Lesson 9-4

Example 1 Factor $ax^2 + bx + c$

a. Factor $4x^2 + 8x - 5$.

In this trinomial, $a = 4$, $b = 8$ and $c = -5$. You need to find two numbers whose sum is 8 and whose product is $4 \cdot -5$ or -20. Make an organized list of the factors of -20, and look for the pair of factors whose sum is 8.

Factors of -20	**Sum of Factors**	
-1, 20	19	
1, -20	-19	
-2, 10	8	The correct factors are –2 and 10.
2, -10	-8	
-4, 5	1	
4, -5	-1	

$4x^2 + 8x - 5 = 4x^2 + mx + nx - 5$ Write the pattern.

$= 4x^2 + -2x + 10x - 5$ $m = -2$ and $n = 10$

$= (4x^2 + -2x) + (10x - 5)$ Group terms with common factors.

$= 2x(2x - 1) + 5(2x - 1)$ Factor the GCF from each grouping.

$= (2x - 1)(2x + 5)$ $2x - 1$ is the common factor.

Check: You can check this result by multiplying the two factors.

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$(2x - 1)(2x + 5) = 4x^2 + 10x - 2x - 5$ FOIL method

$= 4x^2 + 8x - 5$ Simplify.

b. Factor $3x^2 + 11x + 10$.

In this trinomial, $a = 3$, $b = 11$ and $c = 10$. You need to find two numbers whose sum is 11 and whose product is $3 \cdot 10$ or 30. Make an organized list of the factors of 30, and look for the pair of factors whose sum is 11.

Factors of 30	**Sum of Factors**	
1, 30	31	
2, 15	17	
3, 10	13	
5, 6	11	The correct factors are 5 and 6.

$3x^2 + 11x + 10 = 3x^2 + mx + nx + 10$ Write the pattern.

$= 3x^2 + 5x + 6x + 10$ $m = 5$ and $n = 6$

$= (3x^2 + 5x) + (6x + 10)$ Group terms with common factors.

$= x(3x + 5) + 2(3x + 5)$ Factor the GCF from each grouping.

$= (3x + 5)(x + 2)$ Factor out the common factor $3x + 5$.

Example 2 Factor when *a*, *b*, and *c* Have a Common Factor

Factor $36x^2 + 6x - 12$.

Notice that the GCF of the terms $36x^2$, $6x$, and -12 is 6. When the GCF of the terms of a trinomial is an integer other than 1, you should first factor out this GCF.

$36x^2 + 6x - 12 = 6(6x^2 + x - 2)$ Distributive Property

Now factor $6x^2 + x - 2$. You need to find two numbers who sum is 1 and whose product is $6 \cdot -2$ or -12.

Factors of –12	Sum of Factors	
-1, 12	11	
1, -12	-11	
-2, 6	4	
2, -6	-4	
-3, 4	1	The correct factors are -3 and 4.
3, -4	-1	

$6x^2 + x - 2 = 6x^2 + mx + nx - 2$ Write the pattern.

$= 6x^2 - 3x + 4x - 2$ $m = -3$ and $n = 4$

$= (6x^2 - 3x) + (4x - 2)$ Group terms with common factors.

$= 3x(2x - 1) + 2(2x - 1)$ Factor the GCF from each grouping.

$= (2x - 1)(3x + 2)$ Factor out the common factor $2x - 1$.

Thus the complete factorization is $36x^2 + 6x - 12 = 6(2x - 1)(3x + 2)$.

Example 3 Determine Whether a Polynomial is Prime

Factor $3x^2 - x + 1$.

In this trinomial, $a = 3$, $b = -1$ and $c = 1$. Since b is negative, $m + n$ is negative. Since c is positive, mn is positive. So, m and n must be negative. Therefore, make a list of the factors of $3 \cdot 1$ or 3, where both factors are negative. Look for a pair of factors whose sum is –1.

Factors of 3	Sum of Factors
-1, -3	-4

There are no factors whose sum is -1. Therefore, $3x^2 - x + 1$ cannot be factored using integers. Thus, $3x^2 - x + 1$ is a prime polynomial.

Example 4 Solve Equations by Factoring

Solve $k^2 + \frac{8}{3}k = 1$. Check your solutions.

$k^2 + \frac{8}{3}k = 1$	Original equation.
$3(k^2 + \frac{8}{3}k) = 3(1)$	Eliminate fractions by multiplying each side by 3.
$3k^2 + 8k = 3$	Distributive Property
$3k^2 + 8k - 3 = 0$	Rewrite so that one side equals 0.
$(3k - 1)(k + 3) = 0$	Factor the left side.
$3k - 1 = 0$ or $k + 3 = 0$	Zero Product Property
$3k = 1$ $\quad k = -3$	Solve each equation.
$k = \frac{1}{3}$	

The solution set is $\{-3, \frac{1}{3}\}$.

Check:

$$k^2 + \frac{8}{3}k = 1 \qquad\qquad k^2 + \frac{8}{3}k = 1$$

$$(-3)^2 + \frac{8}{3}(-3) \stackrel{?}{=} 1 \qquad\qquad (\frac{1}{3})^2 + \frac{8}{3}(\frac{1}{3}) \stackrel{?}{=} 1$$

$$9 + \frac{-24}{3} \stackrel{?}{=} 1 \qquad\qquad \frac{1}{9} + \frac{8}{9} \stackrel{?}{=} 1$$

$$1 = 1 \checkmark \qquad\qquad 1 = 1 \checkmark$$

Example 5 Solve Real-World Problems by Factoring

A ball is thrown from the top of a building that is 34 feet above ground. How long until the ball is 10 feet above the ground?

Use the model for vertical motion. Let $s = 34$, $v = 40$, and $h = 10$.

$h = -16t^2 + 40t + s$	Vertical motion model
$10 = -16t^2 + 40t + 34$	Substitute.
$0 = -16t^2 + 40t + 24$	Subtract 10 from each side.
$0 = -8(2t^2 - 5t + 3)$	Factor out –8.
$0 = (2t^2 - 5t + 3)$	Divide each side by –8.
$0 = (2t + 1)(t - 3)$	Factor $2t^2 - 5t + 3$.
$2t + 1 = 0$ or $t - 3 = 0$	Zero Product Property
$2t = -1$ $\quad t = 3$	Solve each equation.
$t = -\frac{1}{2}$	

The solutions are $-\frac{1}{2}$ and 3 seconds. The only reasonable solution is the positive 3 seconds, therefore, the ball will reach a height of 10 feet after 3 seconds.