Graphing Systems of Equations

What is a system of equations?

A set of equations, for example, two equations with two unknowns, for which a common solution is sought is called a **system of equations**.

A CHILD'S RIDDLE . . .

Can you think of two numbers that when added together total 7, but when subtracted from each other their difference is 1?

SOLUTION: 4 and 3

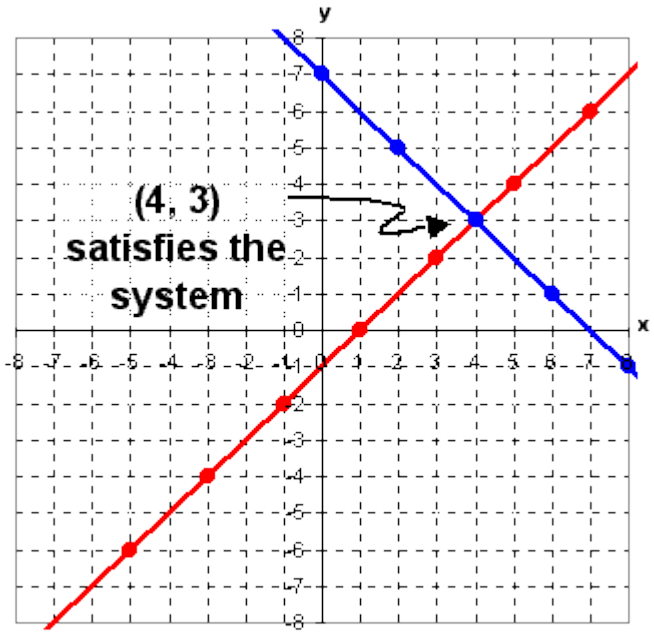
This is just one example of a system of equations. Since the solution of a system must satisfy both conditions simultaneously, we say that we have a **system of simultaneous equations**.

Only two distinct numbers can satisfy this particular system for our riddle. We can show the conditions by graphing.

Example 1:

Graph the system

$$\begin{cases} x + y = 7 \\ x - y = 1 \end{cases}$$



1. You should notice that the x and y values along the blue line satisfy the first condition. That is, the sum of two numbers is 7.

2. Notice the x and y values alone the red line satisfy the second condition. Namely, the difference of two numbers is 1.

3. Since the point (4,3) is the intersection of both lines it satisfies both conditions and is called the solution to the system.

Three Possible Cases:

When we graph a system of two linear equations, one of three things may happen.

1. The lines have one point of intersection. The point of intersection is the ***only solution*** of the system.
2. The lines are parallel. If this is the case, there is no point that satisfies both equations. The system has ***no solution***.
3. The lines coincide. Therefore, the equations have the same graph and every solution of one equation is a solution of the other. There is an ***infinite number of solutions***.

Categorizing Systems by Names – Consistent, Inconsistent, Dependent, and Independent

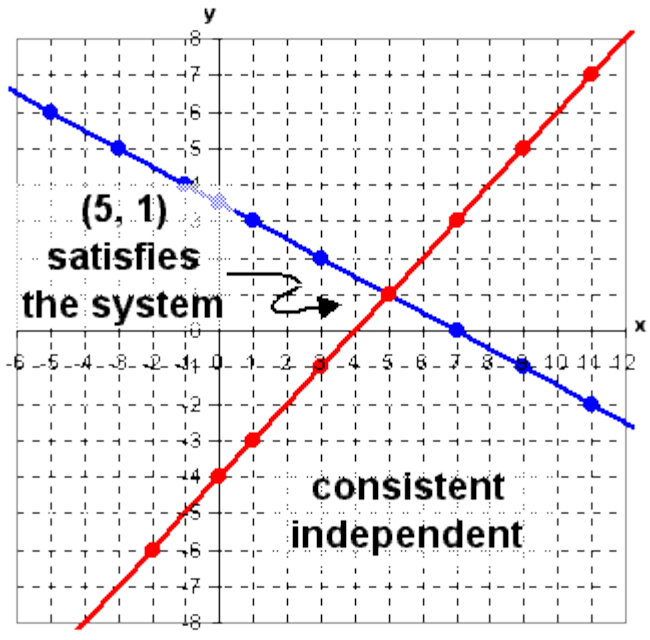
Case:	Number of Solutions:	Name of System
1. lines intersect	one	consistent, independent
2. parallel lines	zero	inconsistent
3. lines coincide	infinitely many	consistent, dependent

More Examples:

Example 2:

Graph the system

$$\begin{cases} x + 2y = 7 \\ x = y + 4 \end{cases}$$

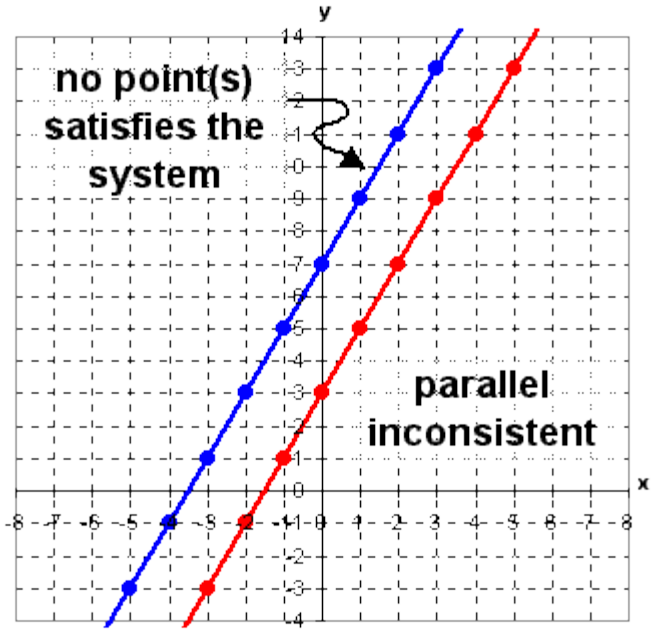


Solve both equations for "y" to graph.
Lines intersect at (5, 1)
System is called consistent and independent.

Example 3:

Graph the system

$$\begin{cases} y - 2x = 7 \\ y = 2x + 3 \end{cases}$$

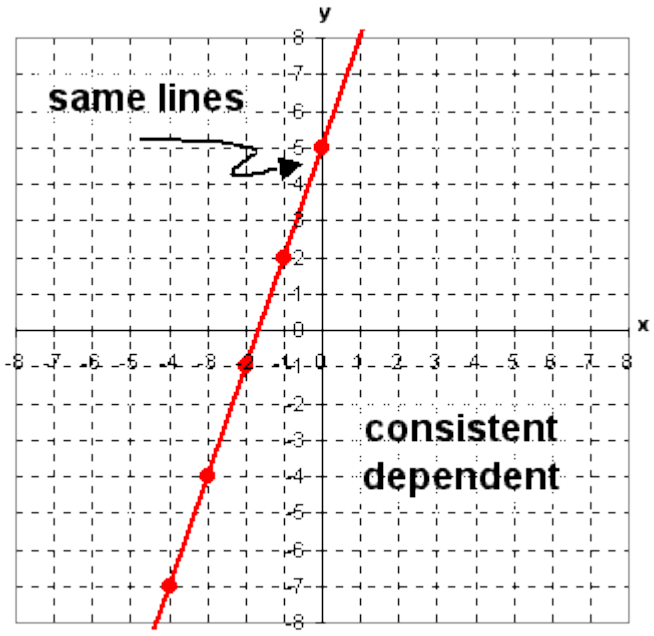


Solve first equation for "y" to graph.
Slopes of both lines are 2, y-intercepts are different.
Lines are parallel, system is called inconsistent.

Example 4

Graph the system

$$\begin{cases} -3x = 5 - y \\ 2y = 6x + 10 \end{cases}$$



Solve both equations for "y" to graph.
Both equations are the graph of y = 3x + 5
Lines coincide; system is called consistent and dependent.