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Study Guide

Student Edition
Pages 4–7

Writing Expressions and Equations

If the cost of one CD is \$15 and you want to buy three CDs, you know that your total cost will be \$15 × 3, or \$45. The expression 15 × 3 is called a **numerical expression**.

You could use a letter such as n to represent the number of CDs you might buy. Then the expression $15 \times n$, or $15n$, would represent the cost of buying n CDs. The expression $15n$ is called an **algebraic expression** because it contains a variable. A variable such as n is a letter used to represent a number. You can use variables to write verbal expressions as algebraic expressions.

Verbal Expressions	Algebraic Expression
2 plus c c more than 2	$2 + c$
k minus 5 k decreased by 5	$k - 5$
two times the product of 2 and x	$2x$
q divided by the sum of 6 and b the quotient of q and the sum of 6 and b	$\frac{q}{6 + b}$

Expressions do not contain equal signs, but tell only which operations to perform. Equations always contain an equal sign.

Example: Write an equation for each sentence.

a. Eight multiplied by 2 equals 16.
$8 \times 2 = 16$

b. Three less than 7 times a number n is 24.
$7n - 3 = 24$

Write an algebraic expression for each verbal expression.

1. the difference of r and 10

2. increase the product of 3 and a by 1

Write a verbal expression for each algebraic expression.

3. $\frac{p}{7}$

4. $2x + 5$

5. $\frac{1}{2}(4 + w)$

Write an equation for each sentence.

6. Seven minus a number z is the same as 15.

7. Ten more than twelve times a number h equals 25.

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Practice

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Pages 4–7

Writing Expressions and Equations

Write an algebraic expression for each verbal expression.

1. the product of 6 and s

2. five less than t

3. g divided by 4

4. 13 increased by y

5. two more than the product of 7 and n

6. the quotient of c and nine decreased by 3

Write a verbal expression for each algebraic expression.

7. $r + 4$

8. $8s$

9. $\frac{t}{5}$

10. $3n - 2$

Write an equation for each sentence.

11. Thirteen decreased by n is equal to 9.

12. Three times g plus five equals 11.

13. Eight is the same as the quotient of 16 and x.

14. Four less than the product of 6 and t is 20.

Write a sentence for each equation.

15. $8 - p = 1$

16. $6x + 3 = 21$

17. $18 \div c = 9$

18. $\frac{2q}{4} = 3$

Enrichment

Perfect, Excessive, Defective, and Amicable Numbers

A **perfect number** is the sum of all of its factors except itself. Here is an example.

$$28 = 1 + 2 + 4 + 7 + 14$$

There are very few perfect numbers. Most numbers are either *defective* or *excessive*.

An **excessive number** is greater than the sum of all of its factors except itself.

A **defective number** is less than this sum.

Two numbers are **amicable** if the sum of the factors of the first number, except for the number itself, equals the second number, and vice versa.

Solve each problem.

1. Write the perfect numbers between 0 and 31.

2. Write the excessive numbers between 0 and 31.

3. Write the defective numbers between 0 and 31.

4. Show that 8128 is a perfect number.

5. The sum of the reciprocals of all the factors of a perfect number (including the number itself) equals 2. Show that this is true for the first two perfect numbers.

6. More than 1000 pairs of amicable numbers have been found. One member of the first pair is 220. Find the other member.

7. One member of the second pair of amicable numbers is 2620. Find the other member.

8. The Greek mathematician Euclid proved that the expression $2^{n-1}(2^n - 1)$ equals a perfect number if the expression inside the parentheses is prime. Use Euclid's expression with n equal to 19 to find the seventh perfect number.

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Order of Operations

Read this sentence: *Jason said Leona is smart.* You need punctuation to tell you whether the sentence means *Jason said, "Leona is smart."* or *"Jason," said Leona, "is smart."*

The meaning of a mathematical expression such as $20 - 2 \times 3$ can also be confusing unless you know which numbers and operations should be grouped together. The order of operations at the right tells you that $20 - 2 \times 3$ means $20 - (2 \times 3)$ or 14.

Order of Operations

1. Find the values of the expressions inside grouping symbols, such as parentheses () and brackets [], and as indicated by fraction bars.
2. Do all multiplications and divisions from left to right.
3. Do all additions and subtractions from left to right.

Example 1: Find the value of $16 \div 8 \times 5$.

$$16 \div 8 \times 5 = 2 \times 5 = 10$$

Multiply and divide from left to right.

Example 2: Find the value of $7(10 - 3)$.

$$7(10 - 3) = 7 \times 7 = 49$$

Simplify within parentheses first.

Example 3: Find the value of $\frac{10 - (2 \times 3)}{5 \div 5}$.

$$\frac{10 - (2 \times 3)}{5 \div 5} = \frac{10 - 6}{5 \div 5}$$

Simplify within parentheses first.

$$= \frac{4}{1}$$

Evaluate the numerator and the denominator separately.

$$= 4$$

Find the value of each expression.

1. $3 + 4 - 2$

2. $6 + 3 \times 7$

3. $1 + 15 \div 5 \times 7$

4. $(7 + 6) \times 5$

5. $2 + 8 \times 3 - 1$

6. $\frac{7 + 1}{2}$

7. $(2 + 8) \times 3 - 1$

8. $\frac{12 + 6}{12 - 6}$

9. $\frac{5 \times 4}{4 + 6}$

10. $5 \times (11 - 7)$

11. $\frac{7 - 5}{10 + 5}$

12. $\frac{1 + 5}{2 \times 9}$

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Practice

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Order of Operations

Find the value of each expression.

1. $16 \div 4 - 3$

2. $6 + 9 \cdot 2$

3. $3(8 - 4) \div 2$

4. $6 \cdot 2 \div 3 + 1$

5. $21 \div [7(12 - 9)]$

6. $\frac{7 + 5}{3 \cdot 2}$

Name the property of equality shown by each statement.

7. $4 + d = 4 + d$

8. If $\frac{y}{3} = 9$ and $y = 27$, then $\frac{27}{3} = 9$.

9. If $3c + 1 = 7$, then $7 = 3c + 1$.

10. If $8 - n = 3 + 1$ and $3 + 1 = 2 \cdot 2$, then $8 - n = 2 \cdot 2$.

Find the value of each expression. Identify the property used in each step.

11. $6(9 - 27 \div 3)$

12. $4(16 \div 16) + 3$

13. $5 + (3 - 6 \div 2)$

14. $8 \div 2 \cdot 7(9 - 8)$

Evaluate each algebraic expression if s = 5 and t = 3.

15. $3(2s - t)$

16. $\frac{4s}{t - 1}$

17. $s + 3t - 8$

18. $s - \frac{t}{3} \cdot 5$

19. $(s + t) - 2 \cdot 3$

20. $3s - 4t + 2$

Enrichment

Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

Reflexive	$a = a$
Symmetric	If $a = b$, then $b = a$.
Transitive	If $a = b$ and $b = c$, then $a = c$.

Other relations have some of the same properties. Consider the relation "is next to" for objects labeled X, Y, and Z. Which of the properties listed above are true for this relation?

X is next to X. *False*
If X is next to Y, then Y is next to X. *True*
If X is next to Y and Y is next to Z, then X is next to Z. *False*

Only the Reflexive Property is true for the relation "is next to."

For each relation, state which properties (Symmetric, Reflexive, Transitive) are true.

1. is the same size as

2. is a family descendant of

3. is in the same room as

4. is the identical twin of

5. is warmer than

6. is on the same line as

7. is a sister of

8. is the same weight as

9. Find two other examples of relations, and tell which properties are true for each relation.

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Study Guide

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Commutative and Associative Properties

Carlos makes salad dressings with olive oil and balsamic vinegar. Sometimes he adds the olive oil first and other times he adds the vinegar first. The salad dressing is always the same, so the order doesn't matter.

The order in which any two numbers are either added or multiplied doesn't change the sum or product. Addition and multiplication are said to be *commutative*.

Commutative Property of Addition	For any two numbers a and b, $a + b = b + a$.
Commutative Property of Multiplication	For any two numbers a and b, $a \cdot b = b \cdot a$.

Example 1: $3x + 4y + 5x$
$= 3x + 5x + 4y$ *Commutative (+)*
$= 8x + 4y$

Example 2: $5 \times 11 \times 2$
$= 5 \times 2 \times 11$ *Commutative (×)*
$= 10 \times 11$
$= 110$

You can also regroup numbers when you are adding or multiplying without changing the sum or product. Addition and multiplication are said to be *associative*.

Associative Property of Addition	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$.
Associative Property of Multiplication	For any numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Example 3: $(10 + 5) + 8$
$= 10 + (5 + 8)$ *Associative (+)*
$= 23$

Example 4: $(100 \cdot 4) \cdot 5$
$= 100 \cdot (4 \cdot 5)$ *Associative (×)*
$= 2000$

Simplify each expression.

1. $(d + 7) + 3$ **2.** $2x + 7 + 5x$ **3.** $2 \times 7k \times 5$

4. $(4a + 2b) + (a + b)$ **5.** $7 \cdot y \cdot 3$ **6.** $(8 \times m) \times 4$

Name the property shown by each statement.

7. $29 + b = b + 29$ **8.** $2(4 \cdot 6) = (2 \cdot 4)6$

9. $(3 + 21) + 7 = (3 + 7) + 21$ **10.** $42 \cdot 8 = 8 \cdot 42$

Commutative and Associative Properties

Name the property shown by each statement.

1. $43 + 28 = 28 + 43$

2. $(9 + 5) + 4 = 9 + (5 + 4)$

3. $(8 \cdot 7) \cdot 11 = 8 \cdot (7 \cdot 11)$

4. $12 \cdot 3 \cdot 6 = 3 \cdot 12 \cdot 6$

5. $(b + 22) + 3 = b + (22 + 3)$

6. $c \cdot d = d \cdot c$

7. $2n + 13 = 13 + 2n$

8. $15 \cdot (2g) = (15 \cdot 2) \cdot g$

Simplify each expression. Identify the properties used in each step.

9. $(m + 7) + 2$

10. $4 \cdot x \cdot 8$

11. $12 + k + 5$

12. $(y \cdot 3) \cdot 12$

13. $13 \cdot (3h)$

14. $7 + 2q + 4$

15. $6n + (9 + 4) + 5$

16. $(7 + p + 22)(9 \div 9)$

17. State whether the statement *Subtraction of whole numbers is associative* is *true* or *false*. If false, provide a counterexample.

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Enrichment

Valid and Faulty Arguments

Consider the statements at the right. What conclusions can you make?

(1) Boots is a cat.
(2) Boots is purring.
(3) A cat purrs if it is happy.

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is *If a cat is happy, then it purrs*.

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin buys Tuff Cote luggage.
Conclusion: Justin's luggage will survive airline travel.

2. (1) If you buy Tuff Cote luggage, it will survive airline travel.
(2) Justin's luggage survived airline travel.
Conclusion: Justin has Tuff Cote luggage.

3. (1) If you use Clear Line long distance service, you will have clear reception.
(2) Anna has clear long distance reception.
Conclusion: Anna uses Clear Line long distance service.

4. (1) If you read the book *Beautiful Braids*, you will be able to make beautiful braids easily.
(2) Nancy read the book *Beautiful Braids*.
Conclusion: Nancy can make beautiful braids easily.

5. (1) If you buy a word processor, you will be able to write letters faster.
(2) Tania bought a word processor.
Conclusion: Tania will be able to write letters faster.

6. (1) Great swimmers wear AquaLine swimwear.
(2) Gina wears AquaLine swimwear.
Conclusion: Gina is a great swimmer.

7. Write an example of faulty logic that you have seen in an advertisement.

Study Guide

Distributive Property

Judi buys a cup of juice and a bagel for Hans and herself at the cafeteria. Juice costs \$1 and a bagel costs \$0.50.

To find the total, Judi finds the total for herself and doubles it. 2(\$1 + \$0.50) = 2 × \$1.50 = \$3	To find the total, Hans finds the cost of 2 bagels and 2 juices and adds them. 2(\$1) + 2(\$0.50) = \$2 + \$1 = \$3

Judi and Hans find the same total.
2(\$1 + \$0.50) = 2(\$1) + 2(\$0.50).
This is an example of the Distributive Property.

Distributive Property

For all numbers a, b, and c,
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$

You can use the Distributive Property to simplify expressions.

Example 1: Simplify $3(x + y) + 4x$.

$$\begin{aligned} 3(x + y) + 4x &= 3x + 3y + 4x && \textit{Distributive Property} \\ &= 3x + 4x + 3y && \textit{Commutative Property} \\ &= 7x + 3y && \textit{Substitution Property} \end{aligned}$$

Example 2: Simplify $7(m + p) + 2(m - p)$.

$$\begin{aligned} 7(m + p) + 2(m - p) &= 7m + 7p + 2m - 2p && \textit{Distributive Property} \\ &= 7m + 2m + 7p - 2p && \textit{Commutative Property} \\ &= 9m + 5p && \textit{Substitution Property} \end{aligned}$$

Simplify each expression.

1. $3(u + v)$

2. $5(k - 2)$

3. $4(2 + 5s) + 3$

4. $7(1 - 2h)$

5. $1(a + 2j + 12k)$

6. $17(c - 2d)$

7. $15(ab + 3c)$

8. $4(2w + 3) + 2$

9. $2(a + 2b) + 3(2a - b)$

10. $7(x + 2y)$

11. $3(e - 4f - ef)$

12. $2(3m + 1) + 4m$

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1–4 Practice

Student Edition
Pages 19–23

Distributive Property

Simplify each expression.

1. $3t + 8t$

2. $7(w + 4)$

3. $8c + 11 - 6c$

4. $2(3n - n)$

5. $5(2r + 3)$

6. $4(6 - 2g)$

7. $15d - 9 + 2d$

8. $(7q + 2z) + (q + 5z)$

9. $24b - b$

10. $6 + 2rs - 5$

11. $9(f + g)$

12. $8x + 2y - 4x - y$

13. $(3a + 2)7$

14. $5(2m - p)$

15. $3(2 - k)$

16. $9(2n + 4)$

17. $12s - 4t + 7t - 3s$

18. $4(2a - 3b)$

19. $(5m + 5n) + (6m - 4n)$

20. $8 + 5z - 6 + z$

21. $2(4x + 3y)$

22. $(hg - 1)7$

23. $13st + 5 - 9st$

24. $8 + 2r + 9$

25. $w + 10 - 4 + 6w$

26. $3(6 + c - 4)$

27. $4(2f - g)$

28. $2 + 7q + 3r + q$

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Enrichment

Student Edition
Pages 19–23

Properties of Operations

Let's make up a new operation and denote it by $\circledast$, so that $a \circledast b$ means b^a.

$2 \circledast 3 = 3^2 = 9$

$(1 \circledast 2) \circledast 3 = 2^1 \circledast 3 = 3^2 = 9$

1. What number is represented by $2 \circledast 3$? __________

2. What number is represented by $3 \circledast 2$? __________

3. Does the operation $\circledast$ appear to be commutative? __________

4. What number is represented by $(2 \circledast 1) \circledast 3$? __________

5. What number is represented by $2 \circledast (1 \circledast 3)$? __________

6. Does the operation $\circledast$ appear to be associative? __________

Let's make up another operation and denote it by $\oplus$, so that $a \oplus b = (a + 1)(b + 1)$.

$3 \oplus 2 = (3 + 1)(2 + 1) = 4 \cdot 3 = 12$

$(1 \oplus 2) \oplus 3 = (2 \cdot 3) \oplus 3 = 6 \oplus 3 = 7 \cdot 4 = 28$

7. What number is represented by $2 \oplus 3$? __________

8. What number is represented by $3 \oplus 2$? __________

9. Does the operation $\oplus$ appear to be commutative? __________

10. What number is represented by $(2 \oplus 3) \oplus 4$? __________

11. What number is represented by $2 \oplus (3 \oplus 4)$? __________

12. Does the operation $\oplus$ appear to be associative? __________

13. What number is represented by $1 \circledast (3 \oplus 2)$? __________

14. What number is represented by $(1 \circledast 3) \oplus (1 \circledast 2)$? __________

15. Does the operation $\circledast$ appear to be distributive over the operation $\oplus$? __________

16. Let's explore these operations a little further. What number is represented by $3 \circledast (4 \oplus 2)$? __________

17. What number is represented by $(3 \circledast 4) \oplus (3 \circledast 2)$? __________

18. Is the operation $\circledast$ actually distributive over the operation $\oplus$? __________

Study Guide

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Pages 24–29

A Plan for Problem Solving

A problem-solving plan can help you identify and organize the information in a problem, then plan and execute a solution. A problem-solving plan should include these steps.

1. **Explore** Read the problem carefully. Identify the information that is given and determine what you need to find.
2. **Plan** Select a strategy for solving the problem. If possible, estimate what you think the answer should be before solving the problem.
3. **Solve** Use your strategy to solve the problem. You may have to choose a variable for the unknown, and then write an expression.
4. **Examine** Check your answer. Does it make sense? Is it reasonably close to your estimate?

Example: A tree in your yard grows 7 inches a year and is now 92 inches tall. In about how many years will the tree be 122 inches tall?

Explore: The tree is already 92 inches tall and it grows 7 inches a year. You need to find how many years it will take the tree to grow to 122 inches.

Plan: Since the tree needs to grow about 30 more inches and $30 \div 7 \approx 4$, you can estimate that it will take more than 4 years for the tree to reach 122 inches.

Solve:

Number of Years From Present	Height of Tree in Inches
1	$92 + 7 = 99$
2	$99 + 7 = 106$
3	$106 + 7 = 113$
4	$113 + 7 = 120$
5	$120 + 7 = 127$

Examine: Your table shows you that the tree will be 122 inches in a little over 4 years. Since the answer matches your estimate, the answer is reasonable.

Solve the problem.

Janine is selling subscriptions to an Internet service. She began by selling one subscription the first day. On the second day she sold two more subscriptions, and on the third day she sold 3 more. If she continues to sell subscriptions according to this pattern, how many will she have sold at the end of one week?

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Practice

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Pages 24–29

A Plan for Problem Solving

Solve each problem. Use any strategy.

1. Tara read 19 science fiction and mystery novels in 6 months. She read 3 more science fiction novels than mystery novels. How many novels of each type did she read?

2. Gasoline costs $1.21 per gallon, tax included. Jaime paid $10.89 for the gasoline he put in his car. How many gallons of gasoline did he buy?

3. A coin-operated telephone at a mall requires 40 cents for a local call. It takes quarters, dimes, and nickels and does not give change. How many combinations of coins could be used to make a local call?

4. Together, Jason and Tyler did 147 sit-ups for the physical fitness test in gym. Jason did 11 fewer sit-ups than Tyler. How many sit-ups did each person do?

5. The perimeter P of a square can be found by using the formula $P = 4s$, where s is the length of a side of the square. What is the perimeter of a square with sides of length 19 cm?

6. Mrs. Hernandez wants to put a picture of each of her 3 grandchildren on a shelf above her desk. In how many ways can she line up the pictures?

7. Leona is 12 years old, and her sister Vicki is 2 years old. How old will each of them be when Leona is twice as old as Vicki?

8. Gunther paid for 6 CDs at a special 2-for-1 sale. The CDs that he got at the sale brought the total number of CDs in his collection to 42. How many CDs did he have before the sale?

9. Phil, Ron, and Felix live along a straight country road. Phil lives 3 miles from Ron and 4 miles from Felix. Felix lives closer to Ron than he does to Phil. How far from Ron does Felix live?

10. Gere has 3 times as many shirts with print patterns as he does shirts in solid colors. He has a total of 16 shirts. How many shirts in print patterns does he have?

Enrichment

Formulas

Some consumers use the following formula when purchasing a new car.

$d = 0.2(4s + c)$ where d = the price the dealer paid the factory for the new car
s = the sticker price (the factory's suggested price)
and c = the cost for dealer preparation and shipping

1. What is the dealer cost for a car with a sticker price of $12,000 and costs for preparation and shipping of $320?

2. According to this formula, how much money does a dealer make if a car is sold for its sticker price of $9500 and the dealer pays $280 for shipping and preparation?

In *The 1978 Bill James Baseball Abstract,* the author introduced the "runs created" formula.

$R = \frac{(h + w)t}{(b + w)}$ where h = a player's number of hits
w = a player's number of walks
t = a player's number of total bases
b = a player's number of at-bats
and R = the approximate number of runs a team scores that are due to this player's actions

3. On June 15, 1983, the Seattle Mariners traded Julio Cruz to the Chicago White Sox for Tony Bernazard. Before the trade, these were the totals for each player.

	h	*w*	*t*	*b*	*runs created*
Cruz	46	20	64	181	________
Bernazard	61	17	87	233	________

Find the number of runs created by each player. Which player created more runs?

4. On August 10, 1983, the New York Yankees traded Jerry Mumphrey to the Houston Astros for Omar Moreno. Before the trade, these were the totals for each player.

	h	*w*	*t*	*b*	*runs created*
Mumphrey	70	28	132	267	________
Moreno	98	8	132	405	________

Find the number of runs created by each player.

Study Guide

Collecting Data

A large ski area surveyed 50 skiers to find out how long they waited in a lift line during a busy period. Their responses are in the chart below.

Time in Lift Line (minutes)									
20	23	20	16	25	26	18	18	19	21
25	24	20	22	19	15	23	22	22	19
18	24	23	16	18	17	17	24	23	23
15	12	22	21	25	24	15	23	24	17
23	22	24	16	16	20	19	23	21	26

A frequency table is one way to organize data so you can draw conclusions more easily from the data. In a frequency table, you use tally marks to record how frequently events occur.

Example: Make a frequency table to organize the survey data. Waiting times vary from 12 minutes to 26 minutes. If you group the waiting times in sets of three, your table will not be too long. The groups are called *intervals*. This table has intervals of three minutes. Each result from the survey is recorded in the tally chart. The total number of tallies is recorded in the Frequency column.

Lift Line Wait		
Time (min)	**Tally**	**Frequency**
12–14	|	1
15–17	𝍸 𝍸	10
18–20	𝍸 𝍸 ||	12
21–23	𝍸 𝍸 𝍸 |	16
24–26	𝍸 𝍸 |	11

Make a frequency table to organize the data in the table. Use intervals of $4.

Profits ($)					
92	95	94	91	100	101
90	90	92	105	101	102
99	94	100	95	93	92
102	103	91	97	105	92

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Practice

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Collecting Data

Determine whether each is a good sample. Describe what caused the bias in each poor sample. Explain.

1. Every third person leaving a music store is asked to name the type of music they prefer.

2. One hundred students at Cary High School are randomly chosen to find the percentage of people who vote in national elections.

3. Two out of 25 students chosen at random in a cafeteria lunch line are surveyed to find whether students prefer sandwiches or pizza for lunch.

Refer to the following chart.

Favorite Leisure Activity									
S	R	C	C	S	R	R	C	S	C
M	S	C	C	C	M	C	C	S	R
S	S	R	M	M	C	M	S	C	R

C = computer games, M = movies,
R = reading, S = sports

4. Make a frequency table to organize the data.

5. What is the most popular leisure activity?

6. How many more people chose sports over reading?

7. Does the information in the frequency table support the claim that most people do not get enough exercise? Explain.

Refer to the following chart.

Number of Breakfasts Eaten Per School Week									
0	5	3	2	0	2	1	3	4	2
5	1	3	2	1	3	1	3	4	1
0	2	3	5	5	2	3	4	1	3

8. Make a frequency table to organize the data.

9. How many students eat breakfast fewer than 3 times per week?

10. Should the school consider a campaign to encourage more students to eat breakfast at school? Explain.

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Enrichment

Student Edition
Pages 32–37

Latin Squares

In designing a statistical experiment, it is important to try to randomize the variables. For example, suppose 4 different motor oils are being compared to see which give the best gasoline mileage. An experimenter might then choose 4 different drivers and four different cars. To test-drive all the possible combinations, the experimenter would need 64 test-drives.

To reduce the number of test drives, a statistician might use an arrangement called a **Latin Square**.

For this example, the four motor oils are labeled A, B, C, and D and are arranged as shown. Each oil must appear exactly one time in each row and column of the square.

The drivers are labeled D(1), D(2), D(3), and D(4); the cars are labeled C(1), C(2), C(3), and C(4).

Now, the number of test-drives is just 16, one for each cell of the Latin Square.

	D(1)	D(2)	D(3)	D(4)
C(1)	A	B	C	D
C(2)	B	A	D	C
C(3)	C	D	A	B
C(4)	D	C	B	A

Create two 4-by-4 Latin Squares that are different from the example.

1.

	D(1)	D(2)	D(3)	D(4)
C(1)				
C(2)				
C(3)				
C(4)				

2.

	D(1)	D(2)	D(3)	D(4)
C(1)				
C(2)				
C(3)				
C(4)				

Make three different 3-by-3 Latin Squares.

3.

	D(1)	D(2)	D(3)
C(1)			
C(2)			
C(3)			

4.

	D(1)	D(2)	D(3)
C(1)			
C(2)			
C(3)			

5.

	D(1)	D(2)	D(3)
C(1)			
C(2)			
C(3)			

Study Guide

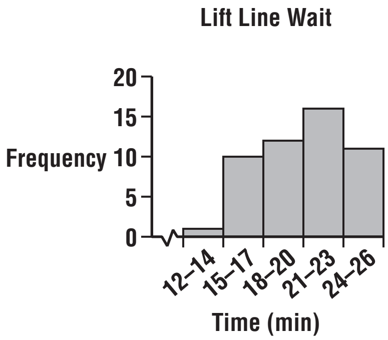
Displaying and Interpreting Data

Data can be easier to analyze if they are presented in the form of a graph. There are many ways to graph data, such as line graphs, histograms, and stem-and-leaf plots. A histogram is a graph of the data in a frequency table.

Example: The frequency table shows the amount of time skiers waited in a lift line. Construct a histogram for the data.

Lift Line Wait		
Time (min)	**Tally**	**Frequency**
12–14	|	1
15–17	~~||||~~ ~~||||~~	10
18–20	~~||||~~ ~~||||~~ ||	12
21–23	~~||||~~ ~~||||~~ ~~||||~~ |	16
24–26	~~||||~~ ~~||||~~ |	11

- The horizontal axis displays the time intervals from the table.
- The vertical axis displays equal intervals of 1.
- For each time interval, draw a bar. The height of the bar is equal to its frequency.
- Label the two axes and title the histogram.



Make a histogram of the data in the frequency table.

Students Visiting Museum		
Ages	**Tally**	**Frequency**
8–9	~~||||~~ |	6
10–11	~~||||~~ ~~||||~~	10
12–13	~~||||~~ ~~||||~~ ~~||||~~	15
14–15	~~||||~~ ~~||||~~ ||	12
16–17	||||	4

NAME ______________________ DATE __________ PERIOD ______

Practice

Student Edition
Pages 38–43

Displaying and Interpreting Data

Use the table below for Exercises 1–4.

Year	U.S. Population
1960	179.3 million
1970	203.3 million
1980	226.5 million
1990	248.7 million

1. Make a line graph of the data. Use the space provided at the right.

2. For which ten-year interval was population growth the greatest?

3. Describe the general trend in the population.

4. Predict the U.S. population for the year 2000.

Use the table at the right for Exercises 5–8. In each age group, 100 people were surveyed.

Country Music Listeners	
Age Group	**Number**
10–19	10
20–29	15
30–39	35
40–49	40
50–59	25

5. Make a histogram of the data.

6. Which age group listens to country music the least?

7. How many respondents in the 40–49 age group listen to country music?

8. Suppose most listeners for a radio station are in their twenties. Should the station play a lot of country music? Explain.

Refer to the stem-and-leaf plot at the right.

Algebra Test Scores

Stem	Leaf
5	6 7 7 8
6	1 4 9
7	3 3 4 5 5 7 8
8	1 3 3 3 6 9
9	0 1 2 4

7 | 5 = 75

9. What were the highest and lowest scores?

10. Which test score occurred most frequently?

11. In which 10-point interval did most of the students score?

12. How many students scored 75 or better?

13. How many students received a score less than 75?

Enrichment

The Digits of π

The number π (pi) is the ratio of the circumference of a circle to its diameter. It is a nonrepeating and nonterminating decimal. No block of the digits of π ever repeats. Here are the first 201 digits of π including 200 digits that follow the decimal point.

3.14159	26535	89793	23846
69399	37510	58209	74944
86280	34825	34211	70679
09384	46095	50582	23172
84102	70193	85211	05559
26433	83279	50288	41971
59230	78164	06286	20899
82148	08651	32823	06647
53594	08128	34111	74502
64462	29489	54930	38196

Solve each problem.

1. If each of the digits appeared with equal frequency, how many times would each digit appear in the first 200 places following the decimal point?

2. Complete this frequency distribution table for the first 200 digits of π that follow the decimal point.

Digit	**Frequency** (Tally Marks)	**Frequency** (Number)	**Cumulative Frequency**
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			

3. Explain how the cumulative frequency column can be used to check a project like this one.

4. Which digit(s) appears most often?

5. Which digit(s) appears least often?

NAME ______________________ DATE __________ PERIOD ______

School-to-Workplace

Brick Quantities (Mason)

To estimate the number of bricks needed for a job, a bricklayer, or mason, needs to know the length, height, and thickness of the wall to be constructed.

The bricklayer first computes the surface area of the wall in feet.

The result is then multiplied by $\frac{7 \times \text{thickness}}{4}$, where the thickness of the wall is measured in inches. The number just computed is then increased by 4% to take breakage of bricks into account.

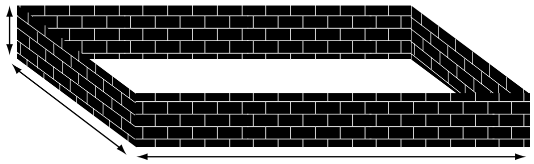
Suppose a wall 6 feet high and 12 inches thick is to be built around a rectangular plot of land 120 feet long and 60 feet wide. Find the number of bricks needed.

Evaluate the following expression.

$$1.04\left[\frac{7 \times 12}{4} \times 6 \times [2(120 + 60)]\right]$$

Use a calculator and follow the correct order of operations.

$$1.04\left[\frac{7 \times 12}{4} \times 6 \times [2(120 + 60)]\right] = 47{,}174.4$$



The bricklayer will need about 47,200 bricks for the wall.

Solve.

1. If the wall described above is to be 8 inches thick rather than 12 inches thick, how many bricks will be needed?

2. How many bricks are needed for a wall 4 feet high and 12 inches thick around a square plot that is 100 feet on a side?

3. How many bricks are needed to build a wall 4 feet high and 9 inches thick around a rectangular plot 100 feet long and 50 feet wide?

4. If the height of a wall to be built is doubled, will the number of bricks needed to build it double?