

Study Guide**Factors**

Because $3 \times 4 = 12$, we say that 3 and 4 are **factors** of 12. In other words, factors are the numbers you multiply to get a product. Since $2 \times 6 = 12$, 2 and 6 are also factors of 12. The only factors of 5 are 1 and 5.

Numbers like 5 that have exactly two factors, the number itself and 1, are called **prime numbers**.

Prime Number	2	3	29
Factors	1, 2	1, 3	1, 29
Products of Factors	1×2	1×3	1×29

Numbers like 12 that have more than two factors are called **composite numbers**.

Composite Number	4	12	33
Factors	1, 2, 4	1, 2, 3, 4, 6, 12	1, 3, 11, 33
Products of Factors	1×4 2×2	1×12 2×6 3×4	1×33 3×11

When two numbers are written as the product of their prime factors, they are in factored form.

Example 1: Write 45 in factored form.

$$\begin{aligned} 45 &= 9 \cdot 5 \\ &= 3 \cdot 3 \cdot 5 \quad \text{Keep factoring until all factors are prime numbers.} \end{aligned}$$

The factored form of 45 is $3 \cdot 3 \cdot 5$.

Example 2: Write $12x^2y$ in factored form.

$$\begin{aligned} 12x^2y &= 3 \cdot 4 \cdot x \cdot x \cdot y \\ &= 3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \end{aligned}$$

The factored form of $12x^2y$ is $3 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y$.

Find the factors of each number. Then classify each number as prime or composite.

1. 10

2. 7

3. 15

4. 21

5. 31

6. 49

7. 47

8. 39

Factor each monomial.

9. $18a$

10. $35xy$

11. c^3

12. $20r^2$

13. $6y^2z$

Practice**Factors**

Find the factors of each number. Then classify each number as prime or composite.

1. 36

2. 31

3. 28

4. 70

5. 43

6. 27

7. 28

8. 97

Factor each monomial.

9. $30m^2n$

10. $-12x^2y^3$

11. $-21ab^2$

12. $36r^3s$

13. $63x^3yz^2$

14. $-40pq^2r^2$

Find the GCF of each set of numbers or monomials.

15. 27, 18

16. 9, 12

17. 45, 56

18. 4, 8, 16

19. 32, 36, 38

20. 24, 36, 48

21. $6x$, $9x$

22. $5y^2$, $15y$

23. $14c^2$, $-13d$

24. $25mn^2$, $20m$

25. $12ab^2$, $18ab$

26. $-28x^2y^3$, $21xy^2$

27. $6xy$, $18y^2$

28. $18c^2d$, $27cd^2$

29. $7m$, mn

Enrichment***Finding the GCF by Euclid's Algorithm***

Finding the greatest common factor of two large numbers can take a long time using prime factorizations. This method can be avoided by using **Euclid's Algorithm** as shown in the following example.

Example: Find the GCF of 209 and 532.

Divide the greater number, 532, by the lesser, 209.

Divide the remainder into the divisor above. Repeat this process until the remainder is zero. The last nonzero remainder is the GCF.	$\begin{array}{r} 2 \\ 209 \overline{)532} \\ \underline{418} \\ 114 \overline{)209} \\ \underline{114} \\ 95 \overline{)114} \\ \underline{95} \\ 19 \overline{)95} \\ \underline{95} \\ 0 \end{array}$
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The divisor, 19, is the GCF of 209 and 532.

Suppose the GCF of two numbers is found to be 1. Then the numbers are said to be **relatively prime**.

Find the GCF of each group of numbers by using Euclid's Algorithm.

- | | |
|---|---|
| 1. 187; 578 | 2. 1802; 106 |
| 3. 161; 943 | 4. 215; 1849 |
| 5. 1325; 3498 | 6. 3484; 5963 |
| 7. 33,583; 4257 | 8. 453; 484 |
| 9. 95; 209; 589 | 10. 518; 407; 851 |
| 11. $17a^2x^2z$; $1615axz^2$ | 12. $752cf^3$; $893c^3f^3$ |
| 13. $979r^2s^2$; $495rs^3$, $154r^3s^3$ | 14. $360x^5y^7$; $328xy$; $568x^3y^3$ |

Study Guide

Factoring Using the Distributive Property

When you use the Distributive Property to multiply a monomial by a polynomial, you show two factors and a product.

Distributive Property	Factors	Product
$3a(5a + 4) = 15a^2 + 12a$	$3a$ and $5a + 4$	$15a^2 + 12a$
$-2x(x^2 + 6x - 1) = -2x^3 - 12x^2 + 2x$	$-2x$ and $x^2 + 6x - 1$	$-2x^3 - 12x^2 + 2x$
$5rs(4r + 2s) = 20r^2s + 10rs^2$	$5rs$ and $4r + 2s$	$20r^2s + 10rs^2$

When you reverse the Distributive Property to identify the factors of the product, the polynomial is said to be in factored form. This is called **factoring** the polynomial.

Example: Factor $15ab^2 + 12a^2b^2$.

$$\begin{array}{l} 15ab^2 = \textcircled{3} \cdot 5 \cdot \textcircled{a} \cdot \textcircled{b} \cdot \textcircled{b} \\ 12a^2b^2 = 2 \cdot 2 \cdot \textcircled{3} \cdot \textcircled{a} \cdot \textcircled{a} \cdot \textcircled{b} \cdot \textcircled{b} \end{array} \quad \begin{array}{l} \text{Begin by factoring each monomial.} \\ \text{Then identify the factors both} \\ \text{monomials have in common.} \end{array}$$

The common factors of $15ab^2$ and $12a^2b^2$ are $3 \cdot a \cdot b \cdot b$, so the greatest common factor is $3ab^2$.

$$\begin{array}{ll} 15ab^2 = 3ab^2(5) & \text{Now write each monomial as a product} \\ 12a^2b^2 = 3ab^2(4a) & \text{of } 3ab^2 \text{ and its other factors.} \end{array}$$

The factored form for $15ab^2 + 12a^2b^2$ is $3ab^2(5 + 4a)$.

Use the Distributive Property to check that the factored form is equivalent to the given polynomial.

$$\text{Check: } 3ab^2(5 + 4a) = 3ab^2(5) + 3ab^2(4a) \text{ or } 15ab^2 + 12a^2b^2 \checkmark$$

Factor each polynomial. If the polynomial cannot be factored, write prime.

1. $12a + 3b$

2. $8w + 6$

3. $15d^2 - 18d$

4. $5c^4 + 2c^2$

5. $12mn^3 - 5n$

6. $4g + 13h^3$

7. $2 - 16x^2$

8. $35xy^3 + 7x^2y$

9. $27pw - 25q$

10. $48c^2d^2 + 36c^2d$

11. $ad + 3x^2 + 9$

12. $7gh^2 + 7g + 14gh$

13. $35a^2 + 15a - 20ab^2$

Factoring Using the Distributive Property

Factor each polynomial. If the polynomial cannot be factored, write prime.

1. $4x + 16$

2. $3y^2 + 12y$

3. $10x + 5x^2y$

4. $7yz + 3x$

5. $15r + 20rs$

6. $14ab + 21a$

7. $9xy - 3xy^2$

8. $12m^2n - 18mn^2$

9. $8ab + 2a^2b^2$

10. $16a^2bc - 36ab^2$

11. $3x^2y + 25m^2$

12. $8x^2y^3 - 10xy$

13. $4xy^2 + 18xy + 14y$

14. $7m^2 + 28mn + 14n^2$

15. $2x^2y + 4xy - 2xy^2$

16. $3a^3b - 9a^2b + 15b^2$

17. $18a^2bc + 24ac^2 + 36a^3c$

18. $8x^3y^2 + 16xy + 28x^2y^3$

Find each quotient.

19. $(6m^2 + 4) \div 2$

20. $(14x^2 - 21x) \div 7x$

21. $(10x^2 + 15y^2) \div 5$

22. $(2c^2 + 4c) \div 2c$

23. $(12xy + 9y) \div 3y$

24. $(9a^2b - 27ab) \div 9ab$

25. $(25m^2n^2 + 15mn) \div 5mn$

26. $(3a^2b - 9abc^2) \div 3ab$

Puzzling Primes

A **prime number** has only two factors, itself and 1. The number 6 is not prime because it has 2 and 3 as factors; 5 and 7 are prime. The number 1 is not considered to be prime.

1. Use a calculator to help you find the 25 prime numbers less than 100.

Prime numbers have interested mathematicians for centuries. They have tried to find expressions that will give all the prime numbers, or only prime numbers. In the 1700s, Euler discovered that the expression $x^2 + x + 41$ will yield prime numbers for values of x from 0 through 39.

2. Find the prime numbers generated by Euler's formula for x from 0 through 7.

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3. Show that the expression $x^2 + x + 31$ will not give prime numbers for very many values of x .

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4. Find the largest prime number generated by Euler's formula. _____

Goldbach's Conjecture is that every nonzero even number greater than 2 can be written as the sum of two primes. No one has ever proved that this is always true. No one has disproved it, either.

5. Show that Goldbach's Conjecture is true for the first 5 even numbers greater than 2.

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6. Give a way that someone could disprove Goldbach's Conjecture.
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Study Guide

Factoring Trinomials: $x^2 + bx + c$

To find the two binomial factors of a polynomial, use the FOIL method.

Example 1: Factor $x^2 + 5x + 6$.

The first term in the trinomial is x^2 . Since $x \cdot x = x^2$, the first term of each binomial is x .

$$x^2 + 5x + 6 = (x + \blacksquare)(x + \blacksquare)$$

To find the last terms, find a number pair whose product is 6 and whose sum is 5.

Product	Factors	Sum
6	1, 6	$1 + 6 = 7$
6	2, 3	$2 + 3 = 5 \checkmark$

Therefore, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

Example 2: Factor $x^2 - 8x + 12$.

The first terms are both x . To find the last terms, find a number pair whose product is 12 and whose sum is -8 .

Product	Factors	Sum
12	$-1, -12$	$-1 + (-12) = -13$
12	$-2, -6$	$-2 + (-6) = -8 \checkmark$
12	$-3, -4$	

Once the correct sum is found, it is not necessary to check any more factors. Therefore, $x^2 - 8x + 12 = (x - 2)(x - 6)$.

Example 3: Factor $x^2 - 2x - 15$.

The first terms are both x . To find the last terms, find a number pair whose product is -15 and whose sum is -2 .

Product	Factors	Sum
-15	$1, -15$	$1 + (-15) = -14$
-15	$-1, 15$	$-1 + 15 = 14$
-15	$3, -5$	$3 + (-5) = -2 \checkmark$

Therefore, $x^2 - 2x - 15 = (x + 3)(x - 5)$.

Factor each trinomial.

1. $x^2 + 3x + 2$

2. $w^2 + 6w + 9$

3. $r^2 + 14r + 24$

4. $z^2 - 6z + 5$

5. $f^2 - 6f + 8$

6. $x^2 - 15x + 56$

7. $v^2 + 15v + 36$

8. $k^2 - 23k + 42$

9. $y^2 - 20y + 100$

10. $a^2 + 4a - 45$

11. $x^2 + 7x - 18$

12. $m^2 - 21m - 22$

Practice**Factoring Trinomials: $x^2 + bx + c$**

Factor each trinomial. If the trinomial cannot be factored, write prime.

1. $x^2 + 5x + 6$

2. $y^2 + 5y + 4$

3. $m^2 + 12m + 35$

4. $p^2 + 8p + 15$

5. $a^2 + 8a + 12$

6. $n^2 + 4n + 4$

7. $x^2 + 9x + 18$

8. $x^2 + x + 3$

9. $y^2 - 6y + 8$

10. $c^2 - 8c + 15$

11. $m^2 - 2m + 1$

12. $b^2 - 9b + 20$

13. $x^2 - 8x + 7$

14. $n^2 - 5n + 6$

15. $y^2 - 8y + 12$

16. $c^2 - 4c + 5$

17. $x^2 - x - 12$

18. $m^2 + 5m - 6$

19. $a^2 + 4a - 12$

20. $y^2 - y - 6$

21. $b^2 - 3b - 10$

22. $x^2 + 3x - 4$

23. $c^2 + 2c - 15$

24. $2x^2 + 10x + 8$

25. $3y^2 - 15y + 18$

26. $5m^2 - 10m - 40$

27. $3b^2 + 6b - 9$

28. $4n^2 + 12n + 8$

29. $2x^2 + 8x - 24$

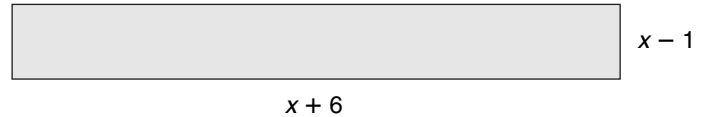
30. $3y^2 - 15y + 12$

Area Models for Quadratic Trinomials

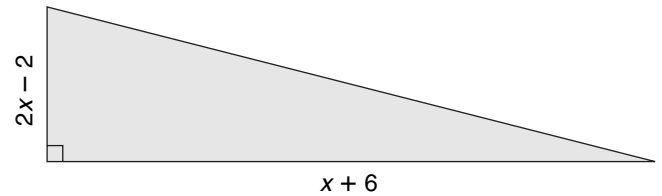
After you have factored a quadratic trinomial, you can use the factors to draw geometric models of the trinomial.

$$x^2 + 5x - 6 = (x - 1)(x + 6)$$

To draw a rectangular model, the value 2 was used for x so that the shorter side would have a length of 1. Then the drawing was done in centimeters. So, the area of the rectangle is $x^2 + 5x - 6$.



To draw a right triangle model, recall that the area of a triangle is one-half the base times the height. So, one of the sides must be twice as long as the shorter side of the rectangular model.



$$\begin{aligned} x^2 + 5x - 6 &= (x - 1)(x + 6) \\ &= \frac{1}{2}(2x - 2)(x + 6) \end{aligned}$$

The area of the right triangle is also $x^2 + 5x - 6$.

Factor each trinomial. Then follow the directions to draw each model of the trinomial.

1. $x^2 + 2x - 3$

Use $x = 2$. Draw a rectangle in centimeters.

2. $3x^2 + 5x - 2$

Use $x = 1$. Draw a rectangle in centimeters.

3. $x^2 - 4x + 3$

Use $x = 4$. Draw two different right triangles in centimeters.

4. $9x^2 - 9x + 2$

Use $x = 2$. Draw two different right triangles. Use 0.5 centimeter for each unit.

Study Guide

Factoring Trinomials: $ax^2 + bx + c$

To find the two binomial factors of a polynomial, use the FOIL method.

Example 1: Factor $5x^2 + 37x + 14$.

The first term in the trinomial is $5x^2$. The only factors of 5 are 5 and 1, so the first terms of the binomials are $5x$ and x .

$$5x^2 + 37x + 14 = (5x + \blacksquare)(x + \blacksquare)$$

The last term in the trinomial is 14, which has two pairs of factors, 1 and 14, and 2 and 7. Try the factor pairs until you find the one that gives a middle term of $37x$.

First Terms	Last Terms	Binomial Pair	Middle Term	Trinomial
$5x, x$	1, 14	$(5x + 1)(x + 14)$	$x + 70x = 71x$	$5x^2 + 71x + 14$
$5x, x$	14, 1	$(5x + 14)(x + 1)$	$14x + 5x = 19x$	$5x^2 + 19x + 14$
$5x, x$	2, 7	$(5x + 2)(x + 7)$	$2x + 35x = 37x$	$5x^2 + 37x + 14 \checkmark$

Therefore, $5x^2 + 37x + 14 = (5x + 2)(x + 7)$.

Example 2: Factor $6x^2 - 23x + 7$.

There are two possible factor pairs of the first term, $2x$ and $3x$, and $6x$ and x . The last term is positive. The sum of the inside and outside terms is negative. So, the factors of 7 are -1 and -7 . Try the factor pairs until you find the one that gives a middle term of $-23x$.

First Terms	Last Terms	Binomial Pair	Middle Term	Trinomial
$2x, 3x$	$-1, -7$	$(2x - 1)(3x - 7)$	$-3x - 14x = -17x$	$6x^2 - 17x + 7$
$3x, 2x$	$-1, -7$	$(3x - 1)(2x - 7)$	$-2x - 21x = -23x$	$6x^2 - 23x + 7 \checkmark$

Therefore, $6x^2 - 23x + 7 = (3x - 1)(2x - 7)$.

Factor each trinomial.

1. $3x^2 + 4x + 1$

2. $2w^2 + 3w + 1$

3. $2r^2 + 5r + 3$

4. $8z^2 + 14z + 5$

5. $5f^2 + 27f + 10$

6. $2x^2 - 3x + 1$

7. $7v^2 - 10v + 3$

8. $9k^2 - 9k + 2$

9. $4y^2 + 3y - 1$

10. $5a^2 + 6a - 8$

Practice**Factoring Trinomials: $ax^2 + bx + c$**

Factor each trinomial. If the trinomial cannot be factored, write prime.

1. $2y^2 + 8y + 6$

2. $2x^2 + 5x + 2$

3. $3a^2 - 4a - 4$

4. $5m^2 - 4m - 1$

5. $2c^2 + 6c - 8$

6. $4q^2 + 2q + 3$

7. $3x^2 - 13x + 4$

8. $4y^2 - 14y + 6$

9. $2b^2 - b - 10$

10. $6a^2 + 8a + 2$

11. $3n^2 + 7n - 6$

12. $3x^2 - 3x - 6$

13. $2c^2 + 3c - 7$

14. $5y^2 - 17y + 6$

15. $2b^2 + 2b - 12$

16. $2x^2 + 10x + 8$

17. $3m^2 - 19m + 6$

18. $4a^2 + 10a - 6$

19. $7b^2 - 16b + 4$

20. $3y^2 - y - 10$

21. $6c^2 + 11c + 4$

22. $10x^2 - x - 2$

23. $12m^2 - 11m + 2$

24. $9y^2 - 3y - 6$

25. $8b^2 + 12b + 4$

26. $6x^2 + 8x - 8$

27. $4n^2 - 14n + 12$

28. $6x^2 + 18x + 12$

29. $4a^2 + 18a - 10$

30. $9y^2 - 15y + 6$

Enrichment**Factoring Trinomials of Fourth Degree**

Some trinomials of the form $a^4 + a^2b^2 + b^4$ can be written as the difference of two squares and then factored.

Example: Factor $4x^4 - 37x^2y^2 + 9y^4$.

Step 1 Find the square roots of the first and last terms.

$$\sqrt{4x^4} = 2x^2 \qquad \sqrt{9y^4} = 3y^2$$

Step 2 Find twice the product of the square roots.

$$2(2x^2)(3y^2) = 12x^2y^2$$

Step 3 Separate the middle term into two parts. One part is either your answer to Step 2 or its opposite. The other part should be the opposite of a perfect square.

$$-37x^2y^2 = -12x^2y^2 - 25x^2y^2$$

Step 4 Rewrite the trinomial as the difference of two squares and then factor.

$$\begin{aligned} 4x^4 - 37x^2y^2 + 9y^4 &= (4x^4 - 12x^2y^2 + 9y^4) - 25x^2y^2 \\ &= (2x^2 - 3y^2)^2 - 25x^2y^2 \\ &= [(2x^2 - 3y^2) + 5xy][(2x^2 - 3y^2) - 5xy] \\ &= (2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2) \end{aligned}$$

Factor each trinomial.

1. $x^4 + x^2y^2 + y^4$

2. $x^4 + x^2 + 1$

3. $9a^4 - 15a^2 + 1$

4. $16a^4 - 17a^2 + 1$

5. $4a^4 - 13a^2 + 1$

6. $9a^4 + 26a^2b^2 + 25b^4$

7. $4x^4 - 21x^2y^2 + 9y^4$

8. $4a^4 - 29a^2c^2 + 25c^4$

Study Guide**Special Factors**

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

Trinomials like the one above that have two equal binomial factors are called **perfect square trinomials**. Recall that when a number is multiplied by itself, the result is a perfect square. For example, $4 \cdot 4 = 16$, so 16 is a perfect square.

Factoring Perfect Square Trinomials	Symbols: $a^2 + 2ab + b^2 = (a + b)(a + b)$ $a^2 - 2ab + b^2 = (a - b)(a - b)$ Example: $x^2 - 2x + 1 = (x - 1)(x - 1)$
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Studying these properties of $x^2 + 10x + 25$ will help you factor other perfect square trinomials.

1. The first term, x^2 , is a perfect square. $x \cdot x = x^2$
2. The last term, 25, is a perfect square. $5 \cdot 5 = 25$
3. The middle term, $10x$, is twice the product of 5 and x . $2(5x) = 10x$

Example 1: Factor $y^2 - 14y + 49$.

The first term is a perfect square. The last term is a perfect square.
The middle term is twice the product of the first and last terms.

So $y^2 - 14y + 49$ is a perfect square trinomial.

$$y^2 - 14y + 49 = (y - 7)(y - 7) \text{ or } (y - 7)^2$$

When two perfect squares are subtracted, the polynomial is called the **difference of two squares**. The difference of two squares also has a special pair of factors.

Factoring Differences of Squares	Symbols: $a^2 - b^2 = (a - b)(a + b)$ Example: $x^2 - 25 = (x - 5)(x + 5)$
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Example 2: Factor $g^2 - 36$.

$g^2 - 36$ is the difference of two squares.

$$g^2 - 36 = (g - 6)(g + 6)$$

Factor each perfect square trinomial.

- | | |
|----------------------|----------------------|
| 1. $y^2 + 16y + 64$ | 2. $a^2 + 14a + 49$ |
| 3. $25s^2 + 10s + 1$ | 4. $r^2 - 8r + 16$ |
| 5. $p^2 - 20p + 100$ | 6. $36h^2 - 12h + 1$ |
| 7. $4a^2 + 12a + 9$ | 8. $9v^2 + 24v + 16$ |

Factor each difference of squares.

- | | |
|----------------|-----------------|
| 9. $b^2 - 64$ | 10. $k^2 - 4$ |
| 11. $81 - x^2$ | 12. $36m^2 - 1$ |
| 13. $4y^2 - 9$ | 14. $4 - 25d^2$ |

Special Factors

Determine whether each trinomial is a perfect square trinomial.

If so, factor it.

1. $y^2 + 6y + 9$

2. $x^2 - 4x + 4$

3. $n^2 + 6n + 3$

4. $m^2 - 12m + 36$

5. $y^2 - 10y + 20$

6. $4a^2 + 16a + 16$

7. $9x^2 + 6x + 1$

8. $4n^2 - 20n + 25$

9. $4y^2 + 9y + 9$

Determine whether each binomial is the difference of squares.

If so, factor it.

10. $x^2 - 49$

11. $b^2 + 16$

12. $y^2 - 81$

13. $4m^2 - 9$

14. $9a^2 - 16$

15. $25r^2 + 9$

16. $18n^2 - 18$

17. $3x^2 - 12y^2$

18. $8m^2 - 18n^2$

Factor each polynomial. If the polynomial cannot be factored, write prime.

19. $4a - 24$

20. $6x + 9$

21. $x^2 + 5x - 10$

22. $2y^2 + 6y - 20$

23. $m^2 - 9n^2$

24. $a^2 - 8a + 16$

25. $5b^2 + 10b$

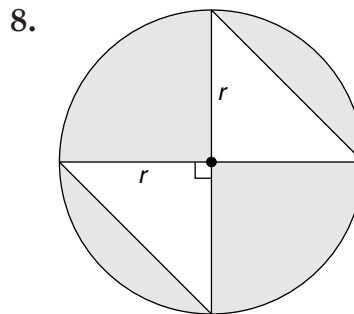
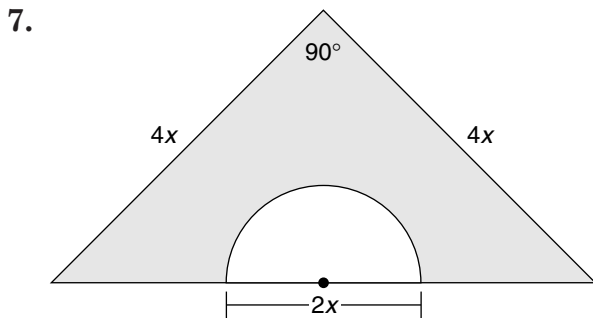
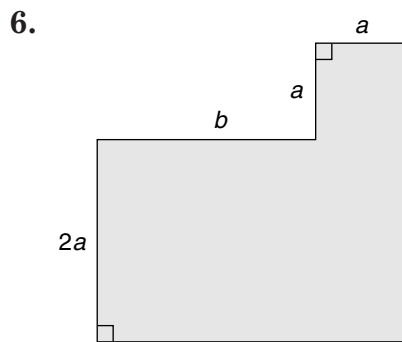
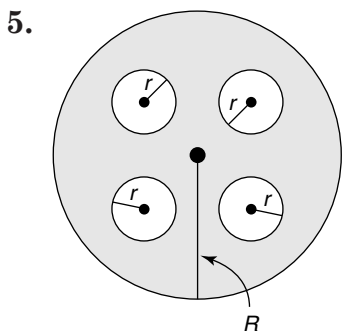
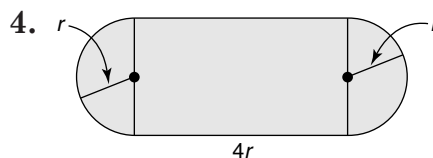
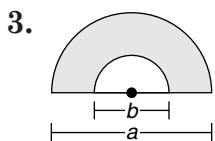
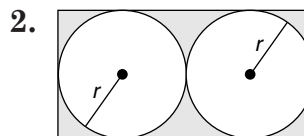
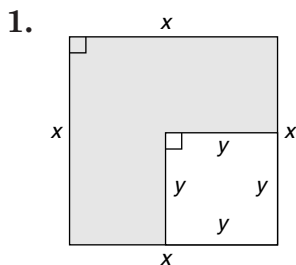
26. $9y^2 + 12y + 4$

27. $3x^2 - 3x - 18$

Enrichment

Writing Expressions of Area in Factored Form

Write an expression in factored form for the area A of the shaded region in each figure below.



Parking Lot Dimensions (Engineer)

Part of the successful design of any office complex is the planning of adequate parking space for employees and visitors. Parking engineering technicians help architects, developers, and builders accomplish this task.

As you might expect, the amount of parking space is variable and depends on the employee capacity of the building. The diagram at the right shows an office building with space adjacent to it designated for parking spaces. One dimension of each parking lot is labeled x because the parking technician may want to experiment with different dimensions.

Suppose the technician has determined that one vehicle needs an 84-square-foot space. Find an expression for the number of cars the lots can accommodate at one time.

Find the area of each lot.

$$\text{Parking Lot A: } x(120 + x)$$

$$\text{Parking Lot B: } (60)x$$

The total area is $x(120 + x) + 60x$.

Factor using the Distributive Property.

$$\begin{aligned} x(120 + x) + 60x &= x(120 + x + 60) \\ &= x(180 + x) \end{aligned}$$

The lots can hold $\frac{x(180 + x)}{84}$ cars at one time.

Solve.

1. Will a choice of $x = 40$ feet make it possible to accommodate 200 cars?
2. Experiment to find the smallest value of x that will make it possible to accommodate 200 cars.
3. If the technician determines that one car needs an 84-square-foot space, find an expression for the number of cars the lots represented at the right can accommodate at one time.

