NAME ______________________ DATE __________ PERIOD ______

11–1 Study Guide

Student Edition
Pages 458–463

Graphing Quadratic Functions

Graphing **quadratic functions** of the form $y = ax^2 + bx + c$, where $a \neq 0$, can be simplified if you know some common characteristics.

Characteristic	Effect
sign of a: $a > 0$ $a < 0$	graph opens upward graph opens downward
axis of symmetry	vertical line at $x = -\frac{b}{2a}$
vertex	maximum or minimum point of the graph; x-coordinate is $-\frac{b}{2a}$

Example: Use characteristics of quadratic functions to graph $y = x^2 - 2x - 1$.

Step 1 First identify a, b, and c in $y = ax^2 + bx + c$: $a = 1$, $b = -2$, and $c = -1$. Since a > 0, the graph opens upwards.

Step 2 Find the axis of symmetry.

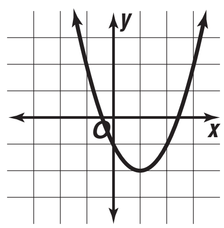
$x = -\frac{b}{2a}$ *Equation of the axis of symmetry*

$x = -\frac{-2}{2(1)}$ or 1 *$a = 1$ and $b = -2$*

Step 3 Find the vertex. Since the equation of the axis of symmetry is $x = 1$, the x-coordinate of the vertex is 1. Substitute 1 into the equation $y = x^2 - 2x - 1$ to get $y = (1)^2 - 2(1) - 1$ or -2. The vertex is at $(1, -2)$.

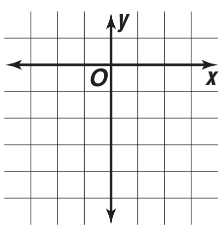
Step 4 Construct a table using values for x that will be on both sides of the axis of symmetry. Choose x-values less than 1 and x-values greater than 1. Graph the points.

x	−1	0	1	2	3
y	2	−1	−2	−1	2

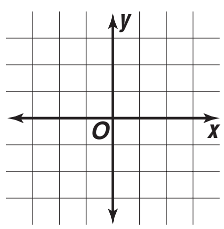


Graph each quadratic equation. Then give the coordinates of the vertex.

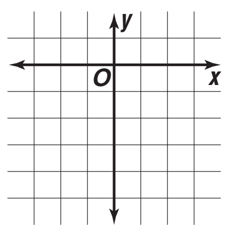
1. $y = -3x^2$



2. $y = x^2 + 2x$



3. $y = -x^2 + 2x - 2$



NAME ______________________ DATE __________ PERIOD ______

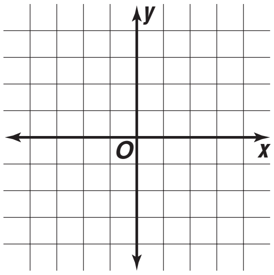
Practice

Student Edition
Pages 458–463

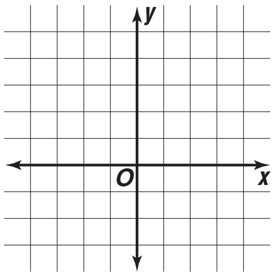
Graphing Quadratic Functions

Graph each quadratic equation by making a table of values.

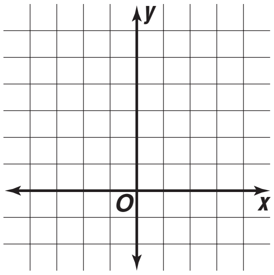
1. $y = x^2 + 2x$



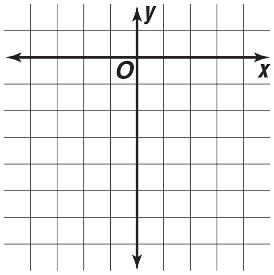
2. $y = -x^2 + 4$



3. $y = -2x^2 + 5$

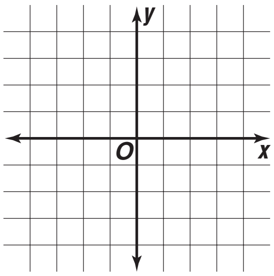


4. $y = x^2 - 2x - 6$

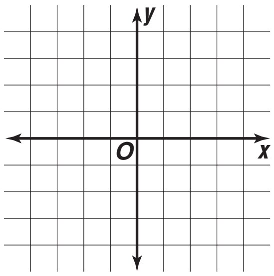


Write the equation of the axis of symmetry and the coordinates of the vertex of the graph of each quadratic function. Then graph the function.

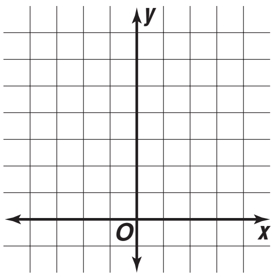
5. $y = x^2 - 1$



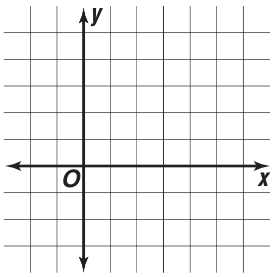
6. $y = x^2 + 4x + 2$



7. $y = -x^2 + 2x + 6$



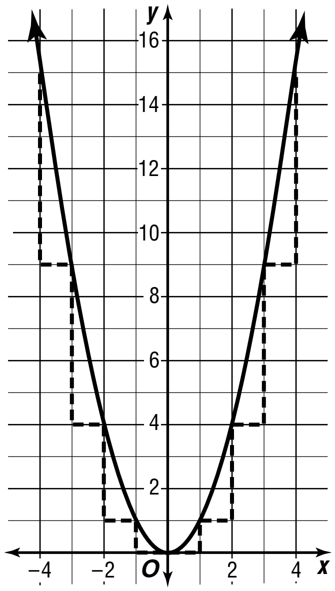
8. $y = -x^2 + 4x$



Enrichment

Odd Numbers and Parabolas

The solid parabola and the dashed stair-step graph are related. The parabola intersects the stair steps at their inside corners.



Use the figure for Exercises 1–3.

1. What is the equation of the parabola?

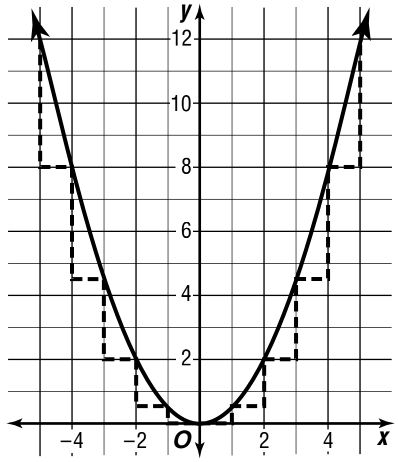
2. Describe the horizontal sections of the stair-step graph.

3. Describe the vertical sections of the stair-step graph.

Use the second figure for Exercises 4–6.

4. What is the equation of the parabola?

5. Describe the horizontal sections of the stair steps.



6. Describe the vertical sections.

7. How does the graph of $y = \frac{1}{2}x^2$ relate to the sequence of numbers $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \cdots$?

8. Complete this conclusion. To graph a parabola with the equation $y = ax^2$, start at the vertex. Then go over 1 and up a; over 1 and up $3a$;

____

____

____

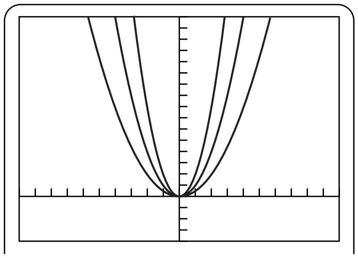
NAME ______________________ DATE __________ PERIOD ______

Study Guide

Families of Quadratic Functions

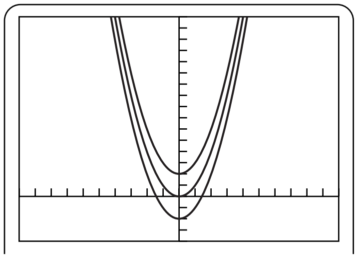
Families of parabolas are parabolas that share the same shape, vertex, or axis of symmetry. Graphing calculators make it easy to study families of graphs. Graph each of the following groups of equations on the same screen to compare and contrast the graphs.

$y = x^2$, $y = 2x^2$, $y = 0.5x^2$



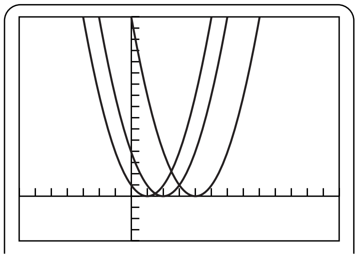
same vertex as $y = x^2$; open upward; different shapes

$y = x^2$, $y = x^2 - 2$, $y = x^2 + 2$



same shape as $y = x^2$; open upward; different vertices

$y = (x - 1)^2$, $y = (x - 2)^2$, $y = (x - 4)^2$



same shape; open upward; shift right

Example: Describe how the graph of $y = (x + 2)^2$ changes from the parent graph of $y = x^2$. Then name the vertex of each graph.

The constant -2 will make the value of the term $(x + 2)^2$ equal to 0. Therefore this graph will shift 2 units to the left. The vertex of $y = x^2$ is at (0, 0), while the vertex of $y = (x + 2)^2$ is at $(-2, 0)$.

Graph each group of equations on the same screen. Compare and contrast the graphs.

1. $y = -x^2$
$y = -4x^2$
$y = -5x^2$

2. $y = (x + 2)^2$
$y = (x + 4)^2$
$y = (x + 6)^2$

3. $y = x^2 - 1$
$y = x^2 - 2$
$y = x^2 - 3$

Describe how each graph changes from the parent graph of $y = x^2$. Then name the vertex of each graph.

4. $y = 4x^2$

5. $y = x^2 - 2$

6. $y = -x^2 - 1$

7. $y = (x + 1)^2$

8. $y = (x - 4)^2$

9. $y = 0.4x^2$

10. $y = x^2 + 3$

11. $y = (x + 3)^2$

12. $y = -x^2 - 5$

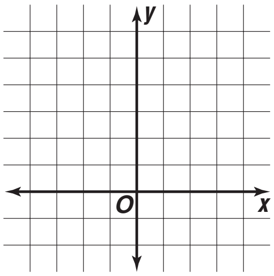
NAME ______________________ DATE __________ PERIOD ______

Practice

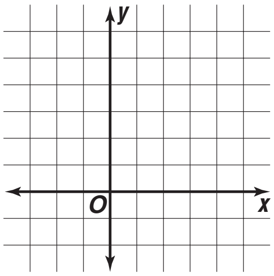
Families of Quadratic Functions

Graph each group of equations on the same axes. Compare and contrast the graphs.

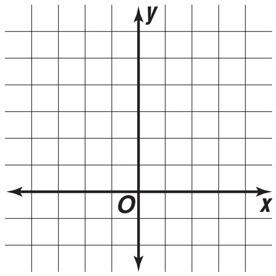
1. $y = -x^2 + 1$
$y = -x^2 + 3$
$y = -x^2 + 5$



2. $y = (x + 1)^2$
$y = (x - 1)^2$
$y = (x - 3)^2$



3. $y = 5.5x^2$
$y = 1.5x^2$
$y = 0.5x^2$



Describe how each graph changes from the parent graph of y = x². Then name the vertex of each graph.

4. $y = 2x^2$

5. $y = x^2 + 3$

6. $y = -x^2 + 5$

7. $y = -0.2x^2$

8. $y = (x + 1)^2$

9. $y = (x - 9)^2$

10. $y = -4x^2 - 1$

11. $y = (x - 6)^2 + 5$

12. $y = -0.5x^2 + 4$

13. $y = 5x^2 + 8$

14. $y = (x - 2)^2 - 3$

15. $y = -(x + 1)^2 + 8$

16. $y = -(x + 3)^2 - 7$

17. $y = -(x - 4)^2 + 5$

18. $y = (x + 6)^2 + 2$

© Glencoe/McGraw-Hill

11–2 **Enrichment**

NAME ______________________ DATE __________ PERIOD ______

Student Edition
Pages 464–467

Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been **translated** to the new position.

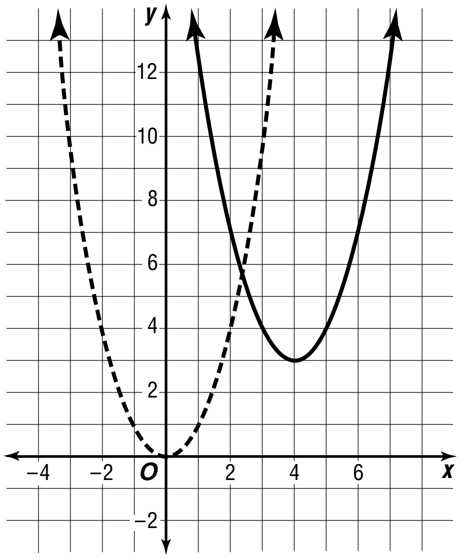
The graph of a quadratic equation in the form $y = (x - b)^2 + c$ is a translation of the graph of $y = x^2$.

Start with $y = x^2$.
Slide to the right 4 units.

$$y = (x - 4)^2$$

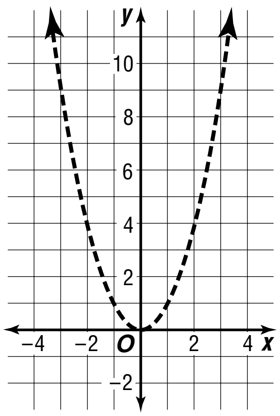
Then slide up 3 units.

$$y = (x - 4)^2 + 3$$

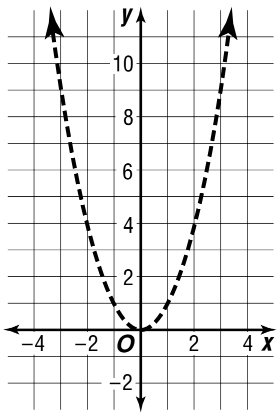


These equations have the form y = x² + c. Graph each equation.

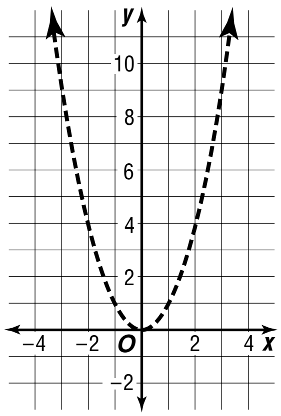
1. $y = x^2 + 1$



2. $y = x^2 + 2$

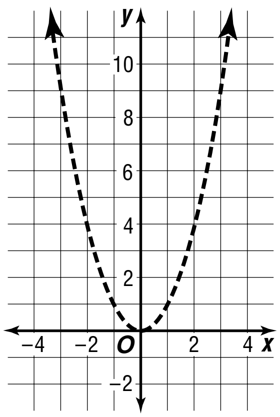


3. $y = x^2 - 2$

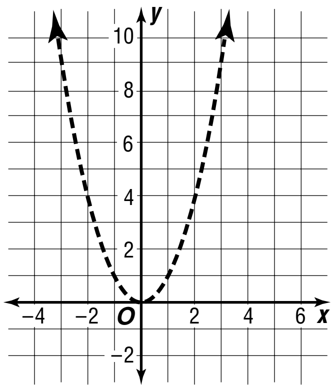


These equations have the form y = (x − b)². Graph each equation.

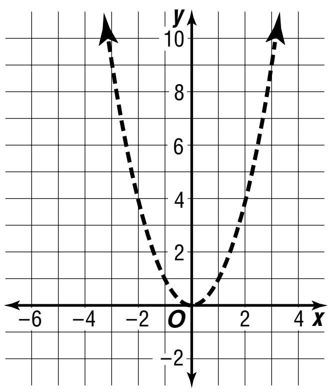
4. $y = (x - 1)^2$



5. $y = (x - 3)^2$



6. $y = (x + 2)^2$



Study Guide

Solving Quadratic Equations by Graphing

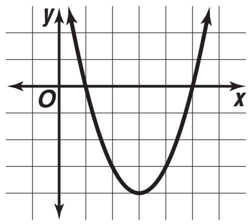
A **quadratic equation** is the equation you get if you set the related quadratic function equal to 0. For example, suppose $h(t) = -16t^2 + 40t + 4$ is the quadratic function representing the height h of a baseball at any time t. A solution to the quadratic equation $0 = -16t^2 + 40t + 4$ represents the time it takes for the ball to hit the ground. The solutions of a quadratic equation are called the **roots** of the equation. They are also the x-intercepts of the related quadratic function.

Example: Find the roots of $x^2 - 6x + 5 = 0$ by graphing the related quadratic function.

Before making a table of values, find the axis of symmetry.

$x = -\frac{b}{2a} = -\frac{-6}{2(1)}$ or 3 $\qquad$ *a = 1 and b = −6*

The equation of the axis of symmetry is $x = 3$. Now make a table using x-values around 3. Graph each point on the coordinate plane.



x	$x^2 - 6x + 5$	**f(x)**
1	$1^2 - 6(1) + 5$	0
2	$2^2 - 6(2) + 5$	−3
3	$3^2 - 6(3) + 5$	−4
4	$4^2 - 6(4) + 5$	−3
5	$5^2 - 6(5) + 5$	0

The x-intercepts of the function are 1 and 5. So the roots are 1 and 5.

Check: Substitute 1 and 5 for x in the equation $x^2 - 6x + 5 = 0$.

$$x^2 - 6x + 5 \stackrel{?}{=} 0 \qquad x^2 - 6x + 5 \stackrel{?}{=} 0$$
$$1^2 - 6(1) + 5 \stackrel{?}{=} 0 \qquad 5^2 - 6(5) + 5 \stackrel{?}{=} 0$$
$$1 - 6 + 5 \stackrel{?}{=} 0 \qquad 25 - 30 + 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

Solve each equation by graphing the related function.

1. $x^2 - 4x + 3 = 0$

2. $x^2 - 2x + 1 = 0$

3. $x^2 + 7x + 6 = 0$

4. $x^2 + 5x - 14 = 0$

5. $x^2 + 10x + 25 = 0$

6. $x^2 - 8x - 9 = 0$

7. $x^2 + 6x = 0$

8. $x^2 + x + 1 = 0$

9. $x^2 + 3x + 2 = 0$

NAME ______________________ DATE __________ PERIOD ______

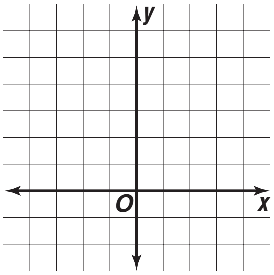
Practice

Student Edition
Pages 468–473

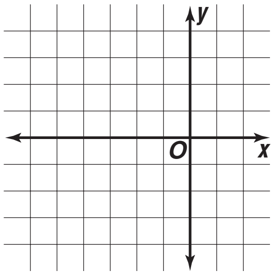
Solving Quadratic Equations by Graphing

Solve each equation by graphing the related function. If exact roots cannot be found, state the consecutive integers between which the roots are located.

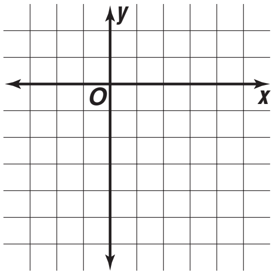
1. $x^2 - 2x + 1 = 0$



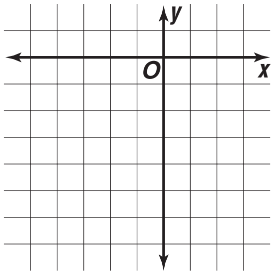
2. $x^2 + 6x + 5 = 0$



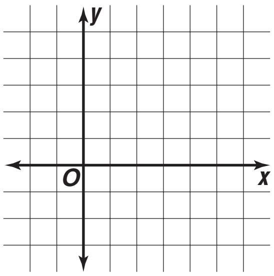
3. $x^2 - 3x - 4 = 0$



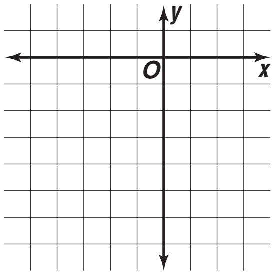
4. $x^2 + 4x - 3 = 0$



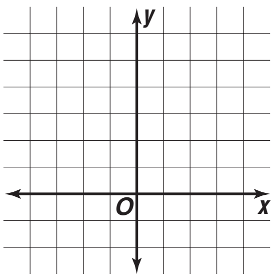
5. $x^2 - 7x + 10 = 0$



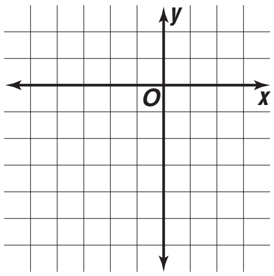
6. $2x^2 - 3x - 6 = 0$



7. $2x^2 - 6x + 3 = 0$



8. $2x^2 + 8x + 2 = 0$



© Glencoe/McGraw-Hill

11–3

NAME ______________________ DATE __________ PERIOD ______

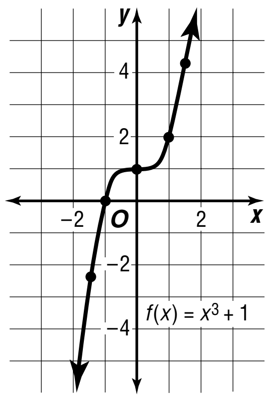
Enrichment

Student Edition
Pages 468–473

Polynomial Functions

Suppose a linear equation such as $-3x + y = 4$ is solved for y. Then an equivalent equation, $y = 3x + 4$, is found. Expressed in this way, y is a function of x, or $f(x) = 3x + 4$. Notice that the right side of the equation is a binomial of degree 1.

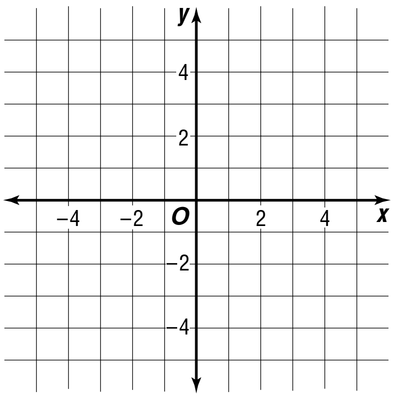
x	y
$-1\frac{1}{2}$	$-2\frac{2}{8}$
-1	0
0	1
1	2
$1\frac{1}{2}$	$4\frac{3}{8}$



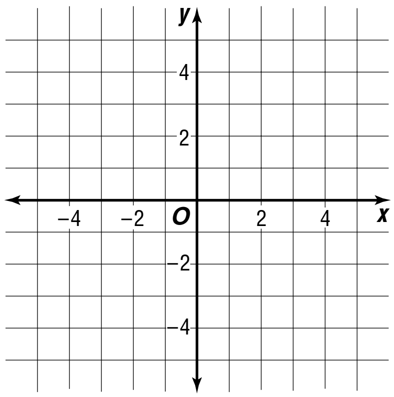
Higher-degree polynomials in x may also form functions. An example is $f(x) = x^3 + 1$, which is a polynomial function of degree 3. You can graph this function using a table of ordered pairs.

For each of the following polynomial functions, make a table of values for x and y = f(x). Then draw the graph on the grid.

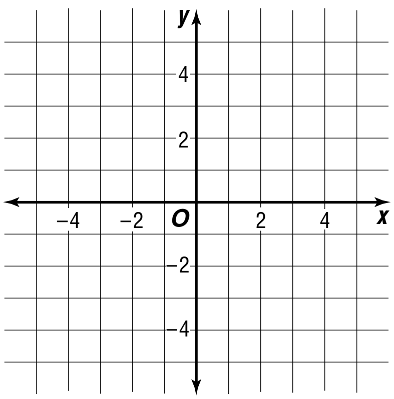
1. $f(x) = 1 - x^2$



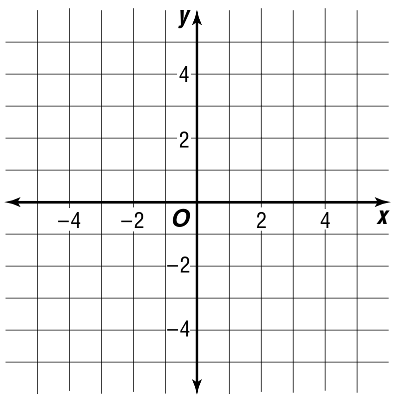
2. $f(x) = x^2 - 5$



3. $f(x) = x^2 + 4x - 1$



4. $f(x) = x^3$



NAME ______________________ DATE __________ PERIOD ______

Study Guide

Student Edition
Pages 474–477

Solving Quadratic Equations by Factoring

The roots of a quadratic equation can be found by factoring trinomial expressions. Using the Zero Product Property shown below, set each factor equal to 0, and then solve each equation.

Zero Product Property	For all numbers a and b, if $ab = 0$, then $a = 0$ or $b = 0$, or both a and $b = 0$.

Example: Solve $x^2 + 7x + 6 = 0$ by factoring.

$x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$ — *Factor.*

$x + 6 = 0$ or $x + 1 = 0$ — *Zero Product Property*

$x = -6$ or $x = -1$ — *Solve each equation.*

The solutions are −6 and −1.

Check: Substitute −6 and −1 for x in the equation $x^2 + 7x + 6 = 0$.

$$\begin{aligned} x^2 + 7x + 6 &= 0 \\ (-6)^2 + 7(-6) + 6 &\stackrel{?}{=} 0 \\ 36 - 42 + 6 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} x^2 + 7x + 6 &= 0 \\ (-1)^2 + 7(-1) + 6 &\stackrel{?}{=} 0 \\ 1 - 7 + 6 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

Solve each equation by factoring. Check your solution.

1. $x^2 + 4x + 3 = 0$

2. $t^2 - 2t + 1 = 0$

3. $x^2 + 5x = 0$

4. $y^2 + 5y - 6 = 0$

5. $p^2 + 12p + 36 = 0$

6. $(k + 2)(k - 3) = 0$

7. $4m(m + 3) = 0$

8. $(g + 2)(g + 7) = 0$

9. $h^2 - h - 2 = 0$

10. $n(n - 3) = 0$

11. $(2g + 2)(g + 4) = 0$

12. $x^2 - 12x = 0$

13. $s^2 - 4s - 12 = 0$

14. $x^2 + 7x + 10 = 0$

15. $y^2 + 16y + 64 = 0$

11–4

NAME ____________ DATE ________ PERIOD ______

Practice

Student Edition
Pages 474–477

Solving Quadratic Equations by Factoring

Solve each equation. Check your solution.

1. $s(s + 3) = 0$

2. $4a(a - 6) = 0$

3. $3m(m + 5) = 0$

4. $6t(t - 2) = 0$

5. $(y + 4)(y - 5) = 0$

6. $(p - 2)(p + 3) = 0$

7. $(x + 5)(x - 6) = 0$

8. $(3r + 2)(r - 1) = 0$

9. $(2n - 2)(n + 1) = 0$

10. $(x - 3)(3x + 6) = 0$

11. $(y + 4)(2y - 8) = 0$

12. $(4c + 3)(c - 7) = 0$

13. $x^2 + 3x - 10 = 0$

14. $x^2 - 6x + 8 = 0$

15. $x^2 + 11x + 30 = 0$

16. $x^2 + 4x = 21$

17. $x^2 - 5x = 36$

18. $x^2 - 5x = 0$

19. $2a^2 = 6a$

20. $2x^2 - 10x + 8 = 0$

21. $3x^2 - 7x - 6 = 0$

22. $5x^2 - x = 4$

23. $3x^2 + 13x = -4$

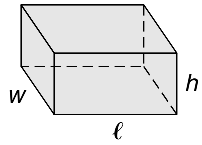
24. $4x^2 + 7x = 2$

© Glencoe/McGraw-Hill

NAME ______________________ DATE __________ PERIOD ______

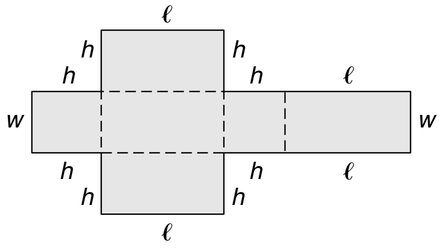
Enrichment

Surface Area of Solid Figures



Many solid objects are formed by rectangles and squares. A box is an example.

The dimensions of the box shown at the right are represented by letters. The length of the base is ℓ units, its width is w units, and the height of the box is h units.



Suppose the box is cut on the seams so that it can be spread out on a flattened surface as shown at the right. The area of this figure is the surface area of the box. Find a formula for the surface area of the box.

There are 6 rectangles in the figure. The surface area is the sum of the areas of the 6 rectangles.

$$S = hw + h\ell + \ell w + h\ell + hw + \ell w$$
$$S = 2\ell w + 2h\ell + 2hw$$

Find the surface area of a box with the given dimensions.

1. $\ell = 14$ cm, $w = 8$ cm, $h = 2$ cm

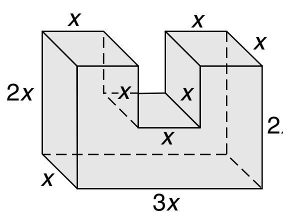
2. $\ell = 40$ cm, $w = 30$ cm, $h = 25$ cm

3. $\ell = x$ cm, $w = (x - 3)$ cm, $h = (x + 3)$ cm

4. $\ell = (s + 9)$ cm, $w = (s - 9)$ cm, $h = (s + 9)$ cm

5. The surface area of a box is 142 cm^2. The length of the base is 2 cm longer than its width. The height of the box is 2 cm less than the width of the base. Find the dimensions of the box.

6. Write an expression that represents the surface area of the figure shown at the right. Include the surface area of the base.



© Glencoe/McGraw-Hill

NAME ______________________ DATE __________ PERIOD ______

Study Guide

Student Edition
Pages 478–482

Solving Quadratic Equations by Completing the Square

Quadratic equations can be solved by **completing the square**. You complete the square for a quadratic expression of the form $x^2 + bx$, according to the steps shown below.

Completing the Square for a Quadratic $x^2 + bx$	**Step 1** Take $\frac{1}{2}$ of b and square it. **Step 2** Add the result to $x^2 + bx$. The perfect square is $x^2 + bx + \left(\frac{b}{2}\right)^2$.	**Example:** For $x^2 + 6x$, $b = 6$. $\frac{1}{2} \times 6 = 3$ $3^2 = 9$ The perfect square is $x^2 + 6x + 9$.

Once you determine $\left(\frac{b}{2}\right)^2$, you must add this number to each side of the equation. Finally, you find the square root of each side to solve the resulting equation.

Example: Solve $x^2 + 8x - 9 = 0$ by completing the square.

$$x^2 + 8x - 9 = 0$$

$$x^2 + 8x = 9 \quad \textit{Add 9 to each side.}$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = 9 + \left(\frac{8}{2}\right)^2 \quad \textit{Add the square of } \tfrac{1}{2}b \textit{ to each side.}$$

$$x^2 + 8x + 16 = 25$$

$$(x + 4)^2 = 25 \quad \textit{Factor } x^2 + 8x + 16.$$

$$x + 4 = \pm\sqrt{25} \quad \textit{Take the square root of each side.}$$

$$x = \pm 5 - 4 \quad \textit{Subtract 4 from each side.}$$

$$x = 5 - 4 \quad \text{or} \quad x = -5 - 4$$

$$x = 1 \quad \text{or} \quad x = -9 \quad \text{The solutions are 1 and } -9.$$

Solve each equation by completing the square.

1. $x^2 + 6x + 9 = 0$

2. $m^2 - 7m - 8 = 0$

3. $y^2 + 4y = 6$

4. $y^2 + 14y = 0$

5. $t^2 - 12t - 7 = 0$

6. $z^2 - 8z - 9 = 0$

7. $n^2 + 5n = 0$

8. $x^2 + 4x = 1$

9. $k^2 + k - 2 = 0$

NAME ____________________ DATE __________ PERIOD ______

Practice

Student Edition
Pages 478–482

Solving Quadratic Equations by Completing the Square

Find the value of c that makes each trinomial a perfect square.

1. $x^2 + 12x + c$

2. $b^2 - 4b + c$

3. $g^2 - 16g + c$

4. $n^2 + 6n + c$

5. $q^2 + 20q + c$

6. $s^2 - 8s + c$

7. $a^2 + 10a + c$

8. $m^2 - 26m + c$

9. $r^2 + 5r + c$

10. $y^2 + y + c$

11. $p^2 - 7p + c$

12. $z^2 + 11z + c$

Solve each equation by completing the square.

13. $x^2 + 10x - 11 = 0$

14. $p^2 - 8p + 12 = 0$

15. $r^2 - 2r - 15 = 0$

16. $c^2 - 4c - 12 = 0$

17. $t^2 - 4t = 0$

18. $x^2 + 6x - 7 = 0$

19. $n^2 + 6n = 16$

20. $w^2 - 14w + 24 = 0$

21. $m^2 - 2m - 5 = 0$

22. $f^2 + 10f + 15 = 0$

23. $s^2 - 6s - 4 = 0$

24. $h^2 - 4h = 2$

25. $y^2 - 12y + 7 = 0$

26. $k^2 - 8k + 13 = 0$

27. $d^2 + 8d + 9 = 0$

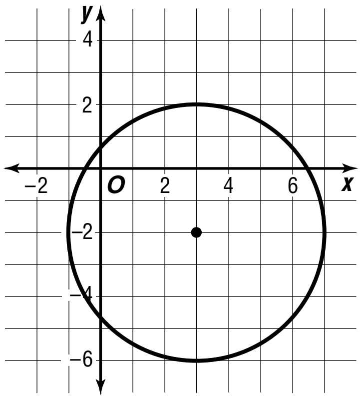
© Glencoe/McGraw-Hill

NAME ______________________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 478–482

Graphing Circles by Completing Squares



One use for completing the square is to graph circles. The general equation for a circle with center at the origin and radius r is $x^2 + y^2 = r^2$. An equation represents a circle if it can be transformed into the sum of two squares.

$$x^2 - 6x + y^2 + 4y - 3 = 0$$
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 0$$
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4$$
$$(x - 3)^2 + (y + 2)^2 = 4^2$$

Notice that the center of the circle is at the point (3, −2).

Transform each equation into the sum of two squares. Then graph the circle represented by the equation. Use the coordinate plane provided at the bottom of the page.

1. $x^2 - 14x + y^2 + 6y + 49 = 0$

2. $x^2 + y^2 - 8y - 9 = 0$

3. $x^2 + 10x + y^2 + 21 = 0$

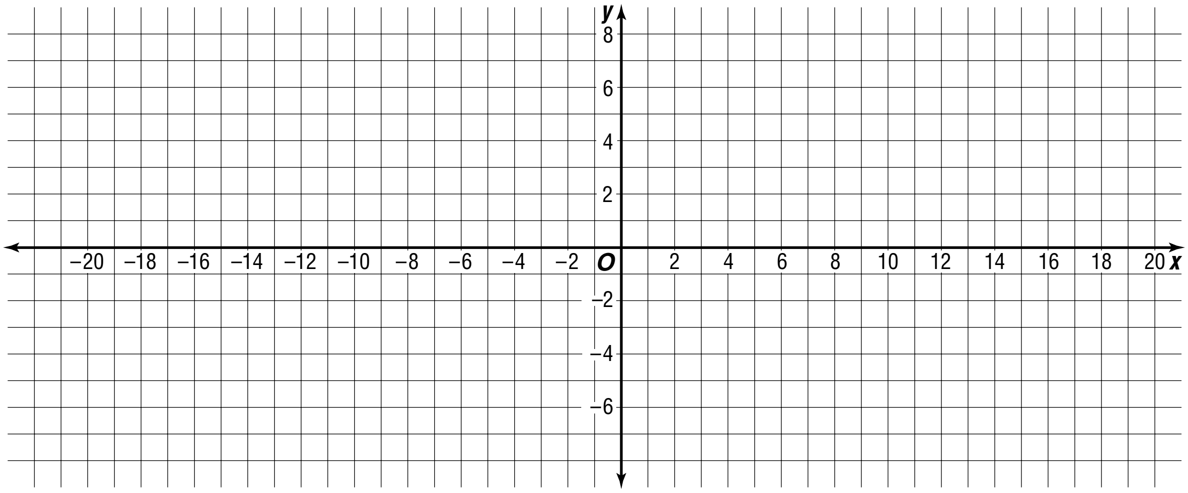
4. $x^2 + y^2 + 10y + 16 = 0$

5. $x^2 - 30x + y^2 + 209 = 0$

6. $x^2 - 18x + y^2 - 12y + 116 = 0$

7. $x^2 + 30x + y^2 - 4y + 193 = 0$

8. $x^2 + 38x + y^2 - 12y + 393 = 0$



NAME ______________________ DATE __________ PERIOD ______

Study Guide

Student Edition
Pages 483–488

The Quadratic Formula

A general formula for solving *any* quadratic equation is the **Quadratic Formula**. Begin with a quadratic equation in the general form $ax^2 + bx + c = 0$, where $a \neq 0$. Identify the values of a, b, and c. Then substitute the values into the Quadratic Formula. If the value of $b^2 - 4ac$ is negative, the equation has no real solutions.

The Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$

Example: Use the Quadratic Formula to solve $2x^2 + 3x - 8 = 0$.

$2x^2 + 3x - 8 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *$a = 2; b = 3; c = -8$*

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-8)}}{2(2)}$ *Substitute the values of a, b, and c into the Quadratic Formula.*

$x = \dfrac{-3 \pm \sqrt{73}}{4}$

The solutions are $\dfrac{-3 + \sqrt{73}}{4}$ and $\dfrac{-3 - \sqrt{73}}{4}$.

Use the Quadratic Formula to solve each equation.

1. $x^2 + 4x + 4 = 0$

2. $n^2 - 8n - 9 = 0$

3. $x^2 + 5x = 8$

4. $m^2 + 18m = 0$

5. $t^2 - 8t + 7 = 0$

6. $k^2 - 10k + 25 = 0$

7. $n^2 - 9n = 0$

8. $2a^2 + a = -6$

9. $m^2 + m - 6 = 0$

10. $n^2 - 6n + 9 = 0$

11. $2y^2 + y = 5$

12. $-x^2 + 9x - 7 = 0$

NAME ______________________ DATE __________ PERIOD ______

Practice

Student Edition
Pages 483–488

The Quadratic Formula

Use the Quadratic Formula to solve each equation.

1. $y^2 - 49 = 0$

2. $x^2 + 7x + 6 = 0$

3. $k^2 - 7k + 12 = 0$

4. $n^2 + 5n - 14 = 0$

5. $b^2 - 5b - 6 = 0$

6. $z^2 + 8z + 12 = 0$

7. $-q^2 + 5q - 4 = 0$

8. $a^2 - 9a + 22 = 0$

9. $c^2 - 4c = -3$

10. $x^2 + 9x = -14$

11. $h^2 - 2h = 8$

12. $m^2 + m = -4$

13. $-z^2 - 8z - 15 = 0$

14. $r^2 + 6r = -5$

15. $-h^2 + 6h = -7$

16. $g^2 + 12x + 20 = 0$

17. $w^2 + 10w = -9$

18. $2y^2 + 6y + 4 = 0$

19. $-2m^2 + 4m + 6 = 0$

20. $2x^2 + 8x = 10$

21. $2b^2 - 3b = -1$

22. $2p^2 + 6p + 8 = 0$

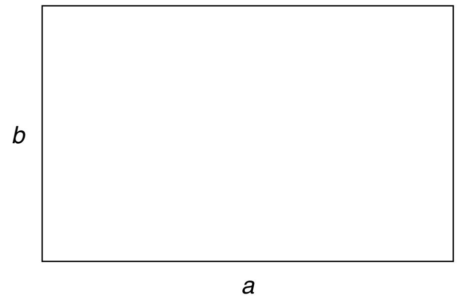
23. $3k^2 + 6k = 9$

24. $-3x^2 - 4x + 4 = 0$

NAME ______________________ DATE __________ PERIOD ______

Enrichment

Golden Rectangles



A **golden rectangle** has the property that its sides satisfy the following proportion.

$$\frac{a+b}{a} = \frac{a}{b}$$

Two quadratic equations can be written from the proportion. These are sometimes called **golden quadratic** equations.

1. In the proportion, let $a = 1$. Use cross-multiplication to write a quadratic equation.

2. Solve the equation in problem 1 for b.

3. In the proportion, let $b = 1$. Write a quadratic equation in a.

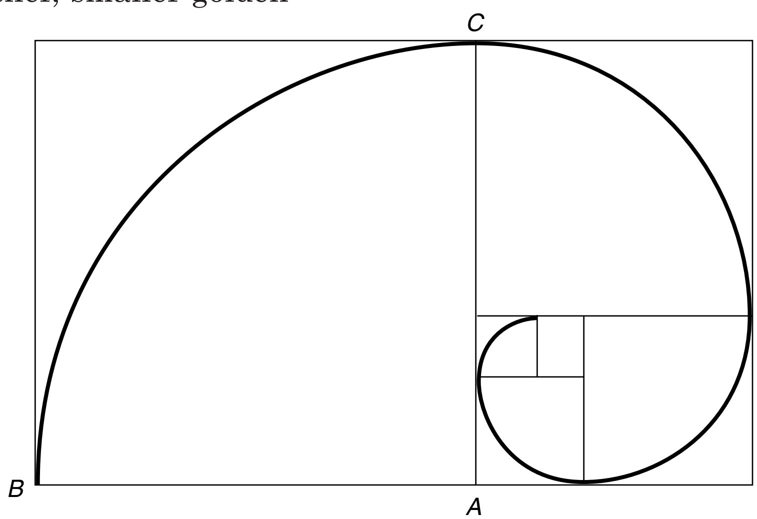
4. Solve the equation in problem 3 for a.

5. Explain why $\frac{1}{2}(\sqrt{5} + 1)$ and $\frac{1}{2}(\sqrt{5} - 1)$ are called golden ratios.

Another property of golden rectangles is that a square drawn inside a golden rectangle creates another, smaller golden rectangle.

In the design at the right, opposite vertices of each square have been connected with quarters of circles.

For example, the arc from point B to point C is created by putting the point of a compass at point A. The radius of the arc is the length BA.



6. On a separate sheet of paper, draw a larger version of the design. Start with a golden rectangle with a long side of 10 inches.

NAME ______________________ DATE __________ PERIOD ______

Study Guide

Student Edition
Pages 489–493

Exponential Functions

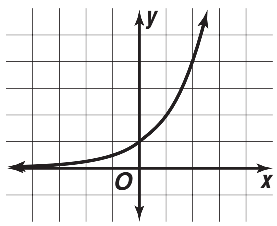
An **exponential function** is a function of the form $y = a^x$, where $a > 0$ and $a \neq 1$. If a is a value greater than one, then the function represents *exponential growth*. This can be very rapid growth, as in the case of $y = 2^x$, or less rapid growth as in the case of $y = 1.05^x$. To graph exponential functions, first make a table of ordered pairs.

x	2^x	y
-1	2^{-1}	0.5
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8

x	1.05^x	y
-1	1.05^{-1}	0.95
0	1.05^0	1
1	1.05^1	1.05
2	1.05^2	1.1
3	1.05^3	1.16

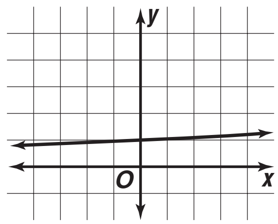
x	2^{2x}	y
-1	2^{-2}	0.25
0	2^0	1
1	2^2	4
2	2^4	16
3	2^6	64

$y = 2^x$



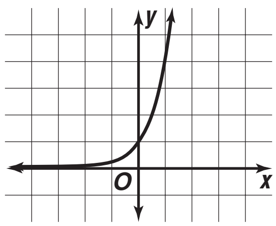
y-intercept = 1

$y = 1.05^x$



y-intercept = 1

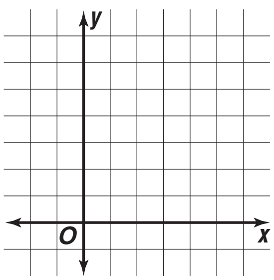
$y = 2^{2x}$



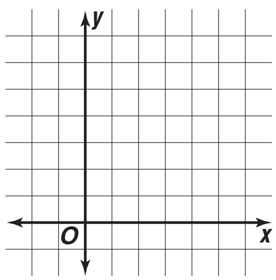
y-intercept = 1

Graph each exponential function.

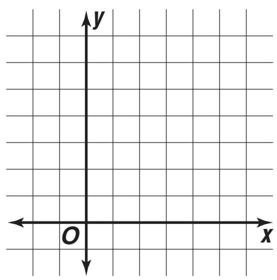
1. $y = 3^x - 1$



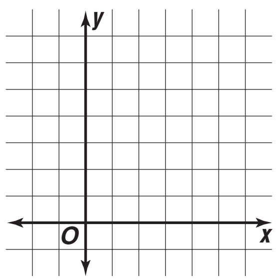
2. $y = 4^x + 2$



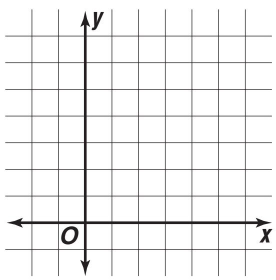
3. $y = 2^x - 1$



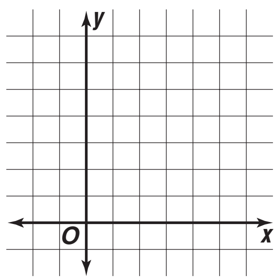
4. $y = 3^{2x}$



5. $y = 4^{2x}$



6. $y = 1.08^x$



NAME ______________________ DATE __________ PERIOD ______

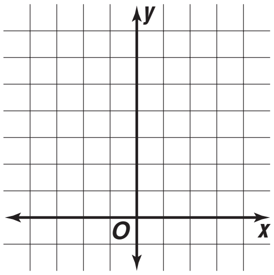
Practice

Student Edition
Pages 489–493

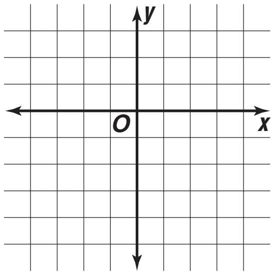
Exponential Functions

Graph each exponential function. Then state the y-intercept.

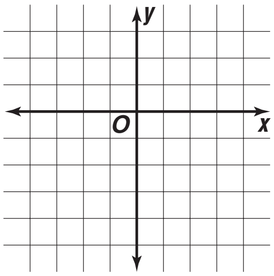
1. $y = 2^x + 3$



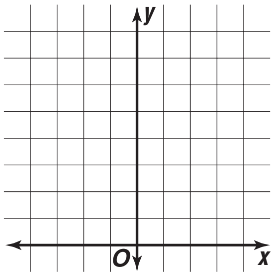
2. $y = 2^x - 2$



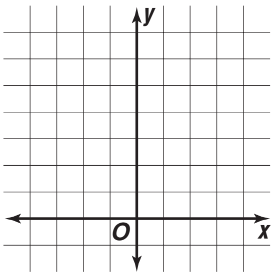
3. $y = 3^x - 4$



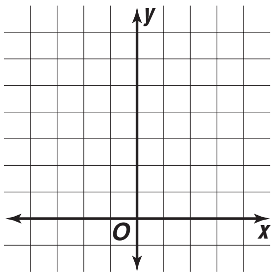
4. $y = 2^x + 4$



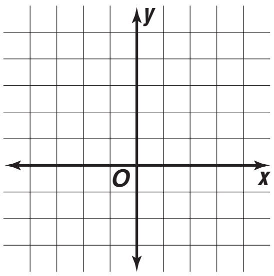
5. $y = 3^x + 1$



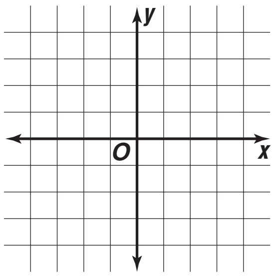
6. $y = 4^x + 2$



7. $y = 3^x - 2$



8. $y = 2^x - 1$



NAME ______________________ DATE __________ PERIOD ______

Enrichment

Student Edition
Pages 489–493

Applications of Geometric Sequence

Populations often grow according to a geometric sequence. If the population of a country grows at the rate of 2% per year, then the common ratio, r, is 1.02. To find the population of a city of 100,000 after 5 years of 2% growth, use the formula ar^{n-1}, where r is the common ratio and n is the number of years.

$$ar^{n-1} = 100{,}000 \times 1.02^{5-1} \qquad a = 100{,}000;\ r = 1.02;\ n = 5$$

$$= 100{,}000 \times 1.02^{4}$$

$$\approx 108{,}243$$

After a few years, a small change in the annual growth rate can cause enormous differences in the population.

Assume that the nation of Grogro had a population of one million in 1990. Using a calculator, find the population of the country in the years 2000, 2050, and 2100 at growth rates of 1%, 3%, and 5% per year. Record your results in the table below.

	Population of Grogro			
	Growth Rate	Year		
		2000	2050	2100
1.	1%			
2.	3%			
3.	5%			

Suppose we want to find the *total* distance a bouncing ball has moved. We need a formula for the *sum* of the terms of a geometric sequence. This is called a **geometric series**.

$$S_n = \frac{a - ar^n}{1 - r}$$

a = *first term*
r = *common ratio* ($r \neq 1$)
n = *number of terms*

Use the formula above to find each sum. Then check your answer by adding.

4. 5 + 10 + 20 + 40 + 80

5. 80 + 240 + 720 + 2160 + 6480

NAME ______________________ DATE __________ PERIOD ______

School-to-Workplace

Student Edition
Pages 468–473

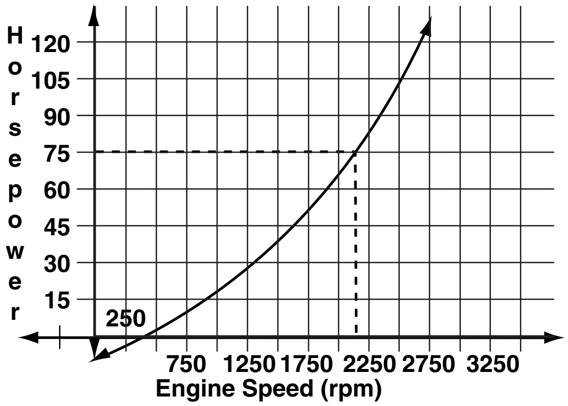
Horsepower (Automotive Technician)

Before the automobile was invented, horses were used to deliver goods from a manufacturer to market, to carry passengers from one place to another, and to raise heavy objects in building projects.

In the late eighteenth century, James Watt estimated that one horse could pull 330 pounds of coal a distance of 100 feet in one minute. So, he defined one horsepower to be 33,000 foot-pounds per minute.

Today's automobiles are high-powered and highly-technical machines that are capable of performing the work of perhaps 165 horses working together.

An automotive technician is trained to understand the relationship between an automobile's horsepower and engine speed measured in crank revolutions per minute.



The equation below relates the horsepower H of an engine to engine rpm r.

$$H = \frac{1}{150{,}000} r^2 + \frac{2}{75} r - \frac{35}{3}$$

At what rpm will the engine generate 75 horsepower?

Graph the equation.

Locate 75 on the vertical axis. From the graph, the desired rpm is between 2000 and 2250. A reasonable estimate for the desired horsepower is $\frac{2000 + 2250}{2} = 2125$.

An engine with an rpm of 2125 will generate about 75 horsepower.

Solve. Give reasonable estimates.

1. At what rpm will the engine generate 60 horsepower?

2. At what rpm will the engine generate 100 horsepower?

3. Part of the graph is below the rpm-axis. What does this tell you about the usefulness of the equation above?