

Study Guide

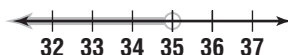
Inequalities and Their Graphs

Many mathematical relationships can be expressed with inequalities. For example, the President of the United States must be at least 35 years old. If a represents his or her age, this can be expressed with the inequality $a \geq 35$. Some verbal phrases that can be used for inequalities are listed in the chart below.

$<$	\leq	$>$	\geq
<ul style="list-style-type: none"> • less than • fewer than 	<ul style="list-style-type: none"> • less than or equal to • at most • no more than • a maximum of 	<ul style="list-style-type: none"> • greater than • more than 	<ul style="list-style-type: none"> • greater than or equal to • at least • no less than • a minimum of

Example 1: Write and graph an inequality to describe the age of people who *cannot* be President of the United States.

Let a represent the age of a person who is less than 35 years old. Then $a < 35$. Since a cannot equal 35, graph a *circle* at 35. Then graph all numbers less than 35 by drawing a line and an arrow to the left.



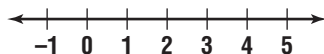
Example 2: Graph $n \geq 2.5$ on a number line.

Since n can equal 2.5, graph a *bullet* at 2.5. Then graph all numbers greater than 2.5 by drawing a line and an arrow to the right.

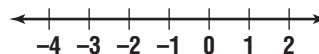


Write an inequality to describe each number. Then graph the inequality on a number line.

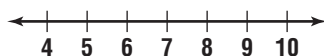
1. a number less than 3



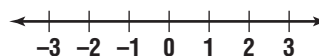
2. a number more than -2



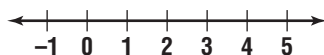
3. a maximum number of 8



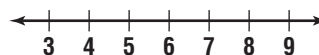
4. a number that is at least -1



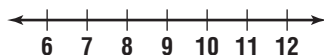
5. a number greater than or equal to $1\frac{1}{2}$



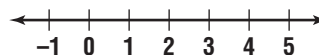
6. a minimum number of $6\frac{1}{5}$



7. a number that is at most 10.2



8. a number less than 2.4



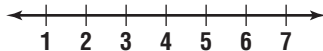
Practice

Inequalities and Their Graphs***Write an inequality to describe each number.***

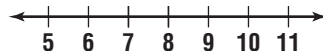
- a number less than or equal to 11
- a number greater than 3
- a number that is at least 6
- a number that is no less than -7
- a maximum number of 9
- a number that is less than -2

Graph each inequality on a number line.

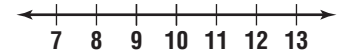
7. $x < 4$



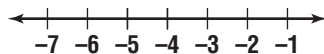
8. $x \geq 8$



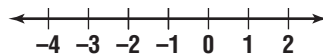
9. $y > 9$



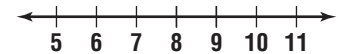
10. $-5 \leq x$



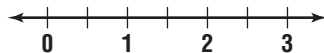
11. $p > -2$



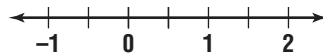
12. $7 \geq g$



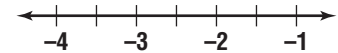
13. $y < 1.5$



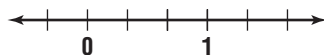
14. $x \geq 0.5$



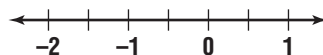
15. $-2.5 > h$



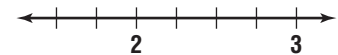
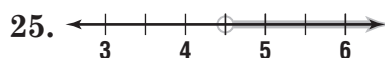
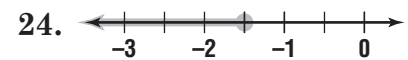
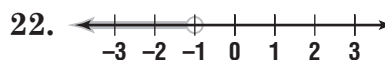
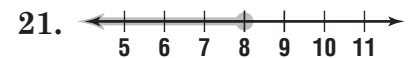
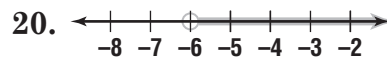
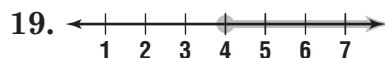
16. $x \leq \frac{1}{3}$



17. $m < -\frac{1}{2}$



18. $2\frac{1}{4} > x$

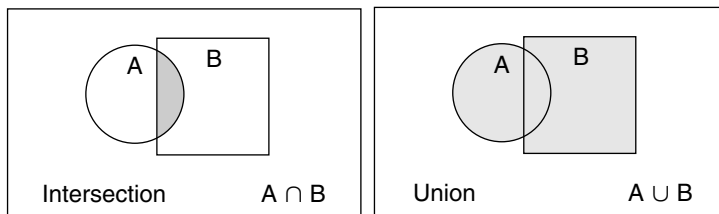
***Write an inequality for each graph.***

Enrichment

Intersection and Union

The **intersection** of two sets is the set of elements that are in both of the sets. The intersection of sets A and B is written $A \cap B$. The **union** of two sets is the set of elements in either A, or in B, or in both. The union is written $A \cup B$.

In the drawings below, suppose A is the set of points inside the circle and B is the set of points inside the square. Then, the shaded areas show the intersection and union.

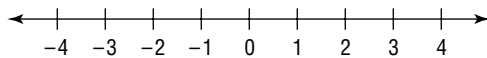


Write $A \cap B$ and $A \cup B$ for each of the following.

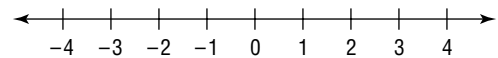
- $A = \{p, q, r, s, t\}$ $B = \{q, r, s\}$ _____
- $A = \{\text{the integers between 2 and 7}\}$ $B = \{0, 3, 8\}$ _____
- $A = \{\text{the states whose names start with K}\}$
 $B = \{\text{the states whose capital is Honolulu or Topeka}\}$ _____
- $A = \{\text{the positive integer factors of 24}\}$
 $B = \{\text{the counting numbers less than 10}\}$ _____

Suppose $A = \{\text{numbers } x \text{ such that } x < 3\}$, $B = \{\text{numbers } x \text{ such that } x \geq -1\}$, and $C = \{\text{numbers } x \text{ such that } x \leq 1.5\}$. Graph each of the following.

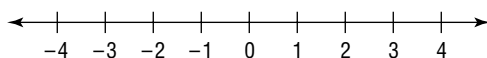
5. $A \cap B$



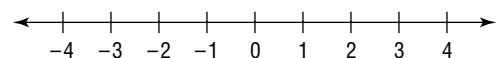
6. $A \cup B$



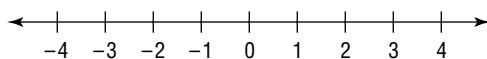
7. $B \cup C$



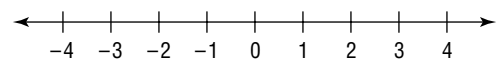
8. $B \cap C$



9. $(A \cap C) \cap B$



10. $A \cap (B \cup C)$



Study Guide

Solving Addition and Subtraction Inequalities

Suppose you already have \$50 and want to earn at least enough money to buy a DVD player for \$325. Let m = the amount of money you earn. You can represent this situation with the inequality $m + 50 \geq 325$. Then the solution to $m + 50 \geq 325$ is the amount of money you must earn. You can use the Addition and Subtraction Properties for Inequalities to solve inequalities involving addition or subtraction. The properties are summarized below.

Addition and Subtraction Properties for Inequalities

For all numbers a , b , and c ,

1. if $a > b$, then $a + c > b + c$, and $a - c > b - c$.
2. if $a \geq b$, then $a + c \geq b + c$, and $a - c \geq b - c$.
3. if $a < b$, then $a + c < b + c$, and $a - c < b - c$.
4. if $a \leq b$, then $a + c \leq b + c$, and $a - c \leq b - c$.

Example: Solve $m + 50 \geq 325$. Check your solution.

$$\begin{aligned} m + 50 &\geq 325 \\ m + 50 - 50 &\geq 325 - 50 && \text{Subtract 50 from each side.} \\ m &\geq 275 \end{aligned}$$

Check: Substitute a number less than 275, the number 275, and a number greater than 275 into the inequality.

Let $m = 200$.	Let $m = 275$.	Let $m = 300$.
$m + 50 \geq 325$	$m + 50 \geq 325$	$m + 50 \geq 325$
$200 + 50 \geq 325$	$275 + 50 \geq 325$	$300 + 50 \geq 325$
$250 \geq 325$; <i>false</i>	$325 \geq 325$; <i>true</i>	$350 \geq 325$; <i>true</i>

In *set-builder notation* the solution is {all numbers greater than or equal to 275}, or $\{m \mid m \geq 275\}$.

Solve each inequality. Check your solution.

- | | |
|------------------------------|---|
| 1. $n + 3 > 6$ | 2. $x - 6 > -2$ |
| 3. $-2 + y \leq 8$ | 4. $x - 4 \leq 12$ |
| 5. $-3 \leq t + 2$ | 6. $1 + p > -1$ |
| 7. $y + 1.2 < 3.4$ | 8. $-2.6 + x > 1.9$ |
| 9. $-1.8 + y \leq 0$ | 10. $x - \frac{1}{2} > \frac{3}{4}$ |
| 11. $1 \leq y - \frac{2}{3}$ | 12. $p - \frac{1}{8} \geq 1\frac{1}{2}$ |

Practice

Solving Addition and Subtraction Inequalities**Solve each inequality. Check your solution.**

1. $x + 7 > 16$

2. $b - 4 < 3$

3. $y - 6 \geq -12$

4. $f + 9 < 24$

5. $a - 2 \leq 9$

6. $3 + w > -1$

7. $n - 1 \leq 7$

8. $10 + c \geq 13$

9. $q - 9 < 4$

10. $-5 \geq d - 7$

11. $17 \geq v + 11$

12. $14 > h - 9$

13. $x + 1.7 \leq 5.8$

14. $2.9 + s < 5.7$

15. $0.3 \leq g - 4.4$

16. $y + \frac{1}{2} \geq 2\frac{3}{4}$

17. $1\frac{1}{4} + m \leq 4\frac{5}{8}$

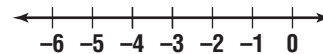
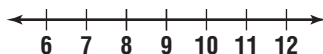
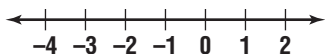
18. $2\frac{1}{6} > r - \frac{2}{3}$

Solve each inequality. Graph the solution.

19. $5x - 2 > 6x$

20. $n + 7 \leq 2n - 1$

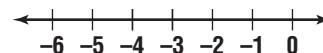
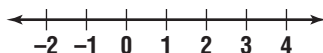
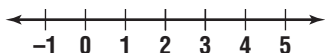
21. $2y + 6 < 3y + 9$



22. $7p \leq 3(2p + 1)$

23. $9m - 6 < 8m - 5$

24. $2h - 11 < 3h - 7$



Consecutive Integers and Inequalities

Consecutive integers follow one after another. For example, 4, 5, 6, and 7 are consecutive integers, as are -8 , -7 , -6 . Each number to the right in the series is one greater than the one that comes before it. If x = the first consecutive integer, then $x + 1$ = the second consecutive integer, $x + 2$ = the third consecutive integer, $x + 3$ = the fourth consecutive integer, and so on.

Example: Find three consecutive positive integers whose sum is less than 12.

$$\begin{array}{ccc} \text{first} & \text{second} & \text{third} \\ \text{integer} & \text{integer} & \text{integer} \\ \hline x & + & x + 1 & + & x + 2 < 12 \end{array}$$

Simplify the expression by combining like terms.

$$3x + 3 < 12$$

$$3x + 3 - 3 < 12 - 3$$

Subtract 3 from each side.

$$3x < 9$$

$$\frac{3x}{3} < \frac{9}{3}$$

Divide each side by 3.

$$x < 3$$

So x could equal 1 or 2.

If $x = 1$, then $x + 1 = 2$, $x + 2 = 3$, and $\{1, 2, 3\}$ is one solution.

If $x = 2$, then $x + 1 = 3$, $x + 2 = 4$, and $\{2, 3, 4\}$ is another solution.

Each of the two solutions must be considered in the answer.

The solution set is $\{1, 2, 3; 2, 3, 4\}$.

Solve. Show all possible solutions.

- Find three consecutive positive integers whose sum is less than 15.
- Find two consecutive positive even integers whose sum is less than 10.
- Find three consecutive positive integers such that the second plus four times the first is less than 21.
- Find three consecutive positive even integers such that the third plus twice the second is less than 26.

Study Guide

Solving Multiplication and Division Inequalities

Suppose the family car gets 25 miles to a gallon of gasoline and you want to calculate how many gallons of gasoline you will need for a trip that is more than 300 miles long. Let g = the number of gallons of gasoline you will need. You can represent this situation with the inequality $25g > 300$. You can use the Multiplication and Division Properties for Inequalities to solve inequalities involving multiplication or division. The properties are summarized below. They are also true for \geq and \leq .

Multiplication and Division Properties for Inequalities

For all numbers a , b , and c ,

1. if c is positive and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
2. if c is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
3. if c is negative and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
4. if c is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Example 1: Solve $25g \geq 300$.

$$\begin{aligned} 25g &\geq 300 \\ \frac{25g}{25} &\geq \frac{300}{25} && \text{Divide by 25.} \\ g &\geq 12 \\ \{g \mid g \geq 12\} \end{aligned}$$

Example 2: Solve $\frac{y}{-2} \leq 8$.

$$\begin{aligned} \frac{y}{-2} &\leq 8 \\ -2\left(\frac{y}{-2}\right) &\geq -2(8) && \text{Multiply by } -2 \\ &&& \text{and reverse the} \\ &&& \text{symbol.} \\ y &\geq -16 \\ \{y \mid y \geq -16\} \end{aligned}$$

Solve each inequality. Check your solution.

1. $3n > 6$

2. $\frac{x}{4} < 18$

3. $-2y \leq 8$

4. $3x > -9$

5. $-8 \leq \frac{t}{-2}$

6. $-2p > -1$

7. $2.4y < -4.8$

8. $-1.5x > 7.2$

9. $6.2y \leq 3.1$

10. $\frac{x}{12} > -3$

11. $\frac{n}{-3} \geq 1.4$

12. $7p > -7$

Practice***Solving Multiplication and Division Inequalities******Solve each inequality. Check your solution.***

1. $4y < 16$

2. $-3q \leq 18$

3. $9g \leq -27$

4. $\frac{p}{5} > 5$

5. $\frac{a}{2} < -4$

6. $-\frac{m}{7} \geq 7$

7. $-6x \leq 30$

8. $-4z > -28$

9. $16 \geq 2e$

10. $-\frac{n}{3} \geq -3$

11. $4 \leq \frac{f}{6}$

12. $-\frac{w}{5} > 8$

13. $-81 < 9v$

14. $6r \leq -42$

15. $-12a \leq -60$

16. $-4 > \frac{u}{9}$

17. $-\frac{d}{6} < -8.1$

18. $\frac{l}{8} > -8$

19. $4k \leq 6$

20. $-0.9b \geq -2.7$

21. $-1.6 < 4t$

22. $\frac{2}{3}y > 6$

23. $-\frac{3}{5}c < 15$

24. $-\frac{5}{8}j \geq -10$

Some Properties of Inequalities

The two expressions on either side of an inequality symbol are sometimes called the *first* and *second* members of the inequality.

If the inequality symbols of two inequalities point in the same direction, the inequalities have the same sense. For example, $a < b$ and $c < d$ have the same sense; $a < b$ and $c > d$ have opposite senses.

In the problems on this page, you will explore some properties of inequalities.

Three of the four statements below are true for all numbers a and b (or a , b , c , and d). Write each statement in algebraic form. If the statement is true for all numbers, prove it. If it is not true, give an example to show that it is false.

1. Given an inequality, a new and equivalent inequality can be created by interchanging the members and reversing the sense.
2. Given an inequality, a new and equivalent inequality can be created by changing the signs of both terms and reversing the sense.
3. Given two inequalities with the same sense, the sum of the corresponding members are members of an equivalent inequality with the same sense.
4. Given two inequalities with the same sense, the difference of the corresponding members are members of an equivalent inequality with the same sense.

Study Guide

Solving Multi-Step Inequalities

Solving inequalities may require more than one operation. The best strategy to use is to undo the operations in reverse order. In other words, first undo addition or subtraction and then undo multiplication or division, just as you did in solving equations with more than one operation. Remember that multiplying or dividing by a negative number reverses the inequality symbol.

Example 1: Solve $6 + 4x \geq 18$.

$$6 + 4x \geq 18$$

$$6 + 4x - 6 \geq 18 - 6$$

$$4x \geq 12$$

$$\frac{4x}{4} \geq \frac{12}{4}$$

$$x \geq 3 \text{ or } \{x \mid x \geq 3\}$$

Subtract 6 from each side.

Divide each side by 4.

Example 2: Solve $4 - 3x < -8 + x$.

$$4 - 3x < -8 + x$$

$$4 - 3x - 4 < -8 + x - 4$$

$$-3x < -12 + x$$

$$-3x - x < -12 + x - x$$

$$-4x < -12$$

$$\frac{-4x}{-4} > \frac{-12}{-4}$$

$$x > 3 \text{ or } \{x \mid x > 3\}$$

Subtract 4 from each side.

Subtract x from each side.

Divide each side by -4 . Reverse the symbol.

Solve each inequality. Check your solution.

1. $2n + 8 > 26$

2. $6x - 12 \leq 48$

3. $-12 - 4y \leq 16$

4. $3x - 1 > -9 - x$

5. $-8 \leq \frac{t}{-2} + 2$

6. $3 \leq 3p + 2$

7. $2 - y < -1.6$

8. $-2x - 8 > 4.2$

9. $y - 3 \leq 2y - 3.1$

10. $3.2x - 16 > -3.2$

11. $6y - 8.2 \leq 36.8$

12. $-1 - 2x < 2$

Solving Multi-Step Inequalities*Solve each inequality. Check your solution.*

1. $3x + 5 < 14$

2. $3t - 6 > 15$

3. $-5y + 2 \geq 32$

4. $-2n - 3 \geq -11$

5. $6 \leq 4a + 10$

6. $-28 < 7 + 7w$

7. $5 - 1.3z \leq 31$

8. $1.7b - 1.1 < 2.3$

9. $6.4 \geq 8 + 2g$

10. $-6 < \frac{k}{2} - 1$

11. $-\frac{c}{6} + 9 \leq 3$

12. $\frac{5m - 5}{3} \geq -15$

13. $\frac{-2n + 6}{4} > 8$

14. $\frac{6 - 3n}{6} \leq -5$

15. $9 - 5j < j - 3$

16. $7p - 4 \geq 3p + 12$

17. $2f - 5 \leq 4f + 13$

18. $5(7 - 2a) \geq -15$

19. $2(q + 2) > 3(q - 6)$

20. $3(h + 5) < -6(h - 4)$

21. $-2(b - 3) \leq 4(b - 9)$

Consecutive Integer Problems

Many types of problems and puzzles involve the idea of consecutive integers. Here is an example.

Find four consecutive odd integers whose sum is -80 .

An odd integer can be written as $2n + 1$, where n is any of the numbers 0, 1, 2, 3, and so on. Then, the equation for the problem is as follows.

$$(2n + 1) + (2n + 3) + (2n + 5) + (2n + 7) = -80$$

Solve these problems. Write an equation or inequality for each.

1. Complete the solution to the problem in the example.
2. Find three consecutive even integers whose sum is 132.
3. Find the two least consecutive integers whose sum is greater than 20.
4. Find the two greatest consecutive integers whose sum is less than 100.
5. The lesser of two consecutive even integers is 10 more than one-half the greater. Find the integers.
6. The greater of two consecutive even integers is 6 less than three times the lesser. Find the integers.
7. Find four consecutive integers such that twice the sum of the two greater integers exceeds three times the first by 91.
8. Find all sets of four consecutive positive integers such that the greatest integer in the set is greater than twice the least integer in the set.

Study Guide

Solving Compound Inequalities

In a doctor's office, you may see a sign that displays the normal weight range for a person of your age and height. For example, if you are a 14-year-old girl who is 5 foot 2 inches tall, it may say that your normal weight is between 100 and 120 pounds, inclusive. Another way to write this information is to use an inequality. If w represents weight, then $100 \leq w \leq 120$ is a **compound inequality** that represents this situation. Another way to write the inequality is to write two inequalities using the word *and*: $100 \leq w$ and $w \leq 120$. A compound inequality using *and* is true if and only if *both* inequalities are true. The graph of a compound inequality using *and* is the **intersection** of the graphs of the inequalities, as shown below.

Example 1: Graph $100 \leq w$ and $w \leq 120$.

Step 1 Graph $100 \leq w$.



Step 2 Graph $w \leq 120$



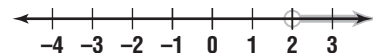
Step 3 Find the intersection of the graphs.



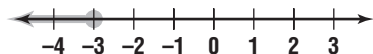
Another type of compound inequality uses the word *or*. A compound inequality using *or* is true if and only if either or both inequalities are true. Its graph is the **union** of the graphs of the inequalities, as shown below.

Example 2: Graph the solution of $x > 2$ or $x \leq -3$.

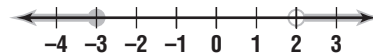
Step 1 Graph $x > 2$.



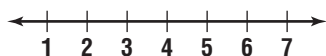
Step 2 Graph $x \leq -3$.



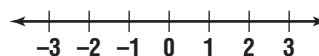
Step 3 Find the union of the graphs.

**Graph the solution of each compound inequality.**

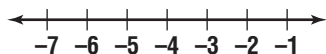
1. $n > 2$ and $n < 6$



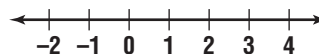
2. $x \leq -2$ or $x > 1$



3. $y \leq -2$ and $y \geq -6$



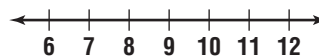
4. $1 \geq p$ and $p > 0$



5. $2 \leq y$ or $y < -1$



6. $h > 8$ and $h \leq 10$



Practice

Solving Compound Inequalities**Write each compound inequality without using and.**

1. $a > 2$ and $a < 7$

2. $b \leq 9$ and $b \geq 6$

3. $w \leq 4$ and $w > -3$

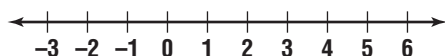
4. $k \geq -4$ and $k < 1$

5. $z < 0$ and $z > -6$

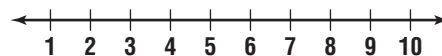
6. $p \geq -8$ and $p < 5$

Graph the solution of each compound inequality.

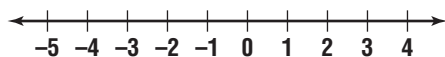
7. $f > -1$ and $f < 5$



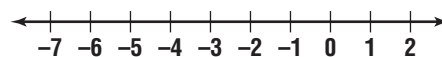
8. $x < 7$ and $x \geq 4$



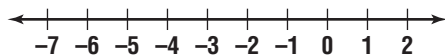
9. $y \leq -3$ or $y \geq 1$



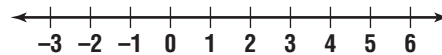
10. $h < -3$ or $h \geq -2$

**Solve each compound inequality. Graph the solution.**

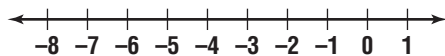
11. $4 > c + 6 \geq 2$



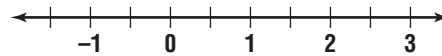
12. $-6 < u - 5 < 0$



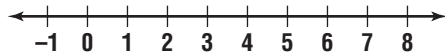
13. $6 < -2m < 10$



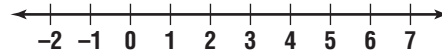
14. $10 > 4n > -2$



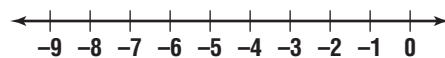
15. $0 \leq \frac{t}{3} \leq 2$



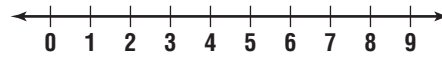
16. $r - 2 < -3$ or $5r > 25$



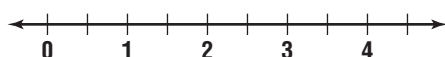
17. $v + 2 \leq -4$ or $v + 7 > 2$



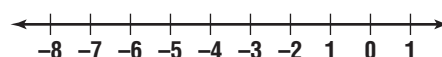
18. $a - 5 < -3$ or $-5a \geq -30$



19. $-4y > -6$ or $2.5y > 5$

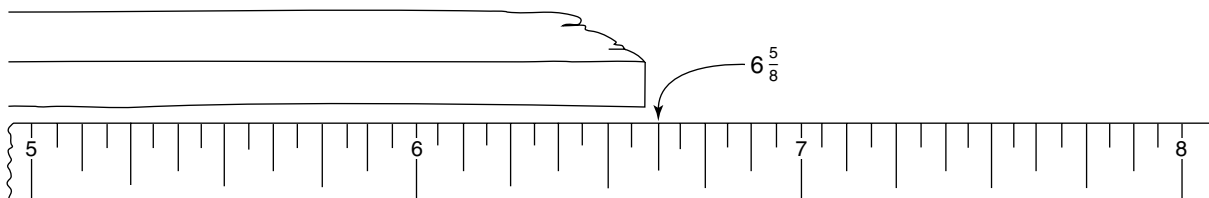


20. $\frac{w}{2} < -1$ or $\frac{w}{3} \leq -2$



Enrichment**Precision of Measurement**

The precision of a measurement depends both on your accuracy in measuring and the number of divisions on the ruler you use. Suppose you measured a length of wood to the nearest one-eighth of an inch and got a length of $6\frac{5}{8}$ in.



The drawing shows that the actual measurement lies somewhere between $6\frac{9}{16}$ in. and $6\frac{11}{16}$ in. This measurement can be written using the symbol \pm , which is read “plus or minus.” It can also be written as a compound inequality.

$$6\frac{5}{8} \pm \frac{1}{16} \text{ in.} \qquad 6\frac{9}{16} \text{ in.} \leq m \leq 6\frac{11}{16} \text{ in.}$$

In this example, $\frac{1}{16}$ in. is the absolute error. The absolute error is one-half the smallest unit used in a measurement.

Write each measurement as a compound inequality. Use the variable m .

1. $3\frac{1}{2} \pm \frac{1}{4}$ in.

2. 9.78 ± 0.005 cm

3. 2.4 ± 0.05 g

4. $28 \pm \frac{1}{2}$ ft

5. 15 ± 0.5 cm

6. $\frac{11}{16} \pm \frac{1}{64}$ in.

For each measurement, give the smallest unit used and the absolute error.

7. $12.5 \text{ cm} \leq m \leq 13.5 \text{ cm}$

8. $12\frac{1}{8} \text{ in.} \leq m \leq 12\frac{3}{8} \text{ in.}$

9. $56\frac{1}{2} \text{ in.} \leq m \leq 57\frac{1}{2} \text{ in.}$

10. $23.05 \text{ mm} \leq m \leq 23.15 \text{ mm}$

Study Guide

Solving Inequalities Involving Absolute Value

You have already studied equations of the form $|x| = n$ involving absolute value, where n is a nonnegative number. Inequalities involving absolute value are similar. They are of the form $|x| > n$ or $|x| < n$, where n is a nonnegative number.

To solve both equations and inequalities involving absolute value, there are two cases to consider.

Case 1 The value of the expression within the absolute value symbol is positive.

Case 2 The value of the expression within the absolute value symbol is negative.

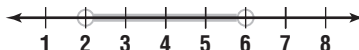
Example 1: Solve $|x - 4| < 2$. Graph the solution.

Case 1 $x - 4$ is positive.

$$\begin{aligned} x - 4 &< 2 \\ x - 4 + 4 &< 2 + 4 \\ x &< 6 \end{aligned}$$

Case 2 $x - 4$ is negative.

$$\begin{aligned} -(x - 4) &< 2 \\ -(x - 4)(-1) &> 2(-1) && \text{Reverse the symbol.} \\ x - 4 &> -2 \\ x - 4 + 4 &> -2 + 4 \\ x &> 2 \end{aligned}$$

The solution is $\{x \mid 2 < x < 6\}$. 

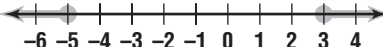
Example 2: Solve $|x + 1| \geq 4$. Graph the solution.

Case 1 $x + 1$ is positive.

$$\begin{aligned} x + 1 &\geq 4 \\ x + 1 - 1 &\geq 4 - 1 \\ x &\geq 3 \end{aligned}$$

Case 2 $x + 1$ is negative.

$$\begin{aligned} -(x + 1) &\geq 4 \\ -(x + 1)(-1) &\leq 4(-1) && \text{Reverse the symbol.} \\ x + 1 &\leq -4 \\ x + 1 - 1 &\leq -4 - 1 \\ x &\leq -5 \end{aligned}$$

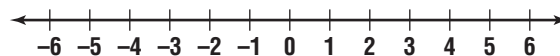
The solution is $\{x \mid x \geq 3 \text{ or } x \leq -5\}$. 

Solve each inequality. Graph the solution.

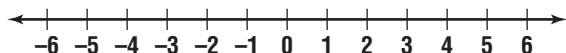
1. $|n| \leq 5$



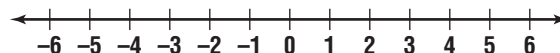
2. $|4x| \leq 12$



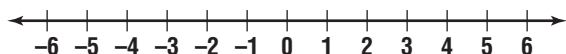
3. $|y + 1| > 2$



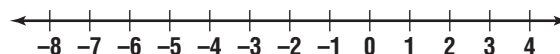
4. $|6p| > 2.4$



5. $|t + 3| \geq 2$



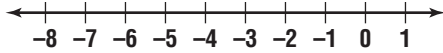
6. $|h + 2| < 6$



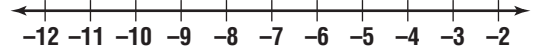
Practice

Solving Inequalities Involving Absolute Value*Solve each inequality. Graph the solution.*

1. $|k + 2| < 1$



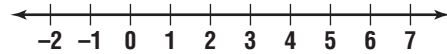
2. $|m + 7| \leq 4$



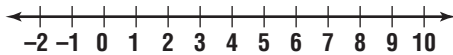
3. $|4p| < 16$



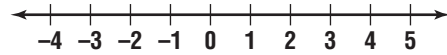
4. $|w - 3| < 3$



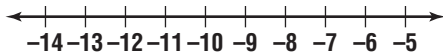
5. $|a - 5| \leq 4$



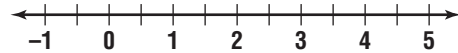
6. $|6t| < 12$



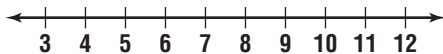
7. $|v + 9| \leq 3$



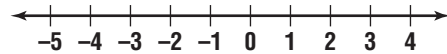
8. $|q - 2| < 2.5$



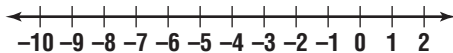
9. $|b - 8| > 2$



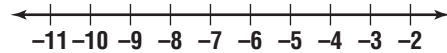
10. $|y + 1| \geq 3$



11. $|x + 4| \geq 4$



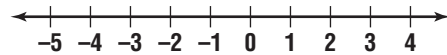
12. $|z + 7| > 2$



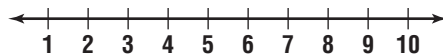
13. $|5c| > 25$



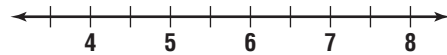
14. $|2g| \geq 2$



15. $|f - 5| \geq 2$



16. $|s - 6| > 1.5$



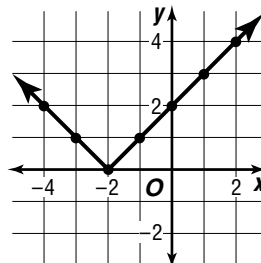
Enrichment

Absolute Value Functions

Some types of functions that occur frequently have special names. Absolute value functions are an example.

Example: Graph $y = |x + 2|$.

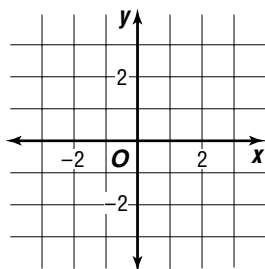
x	y
-4	2
-3	1
-2	0
-1	1
0	2
1	3
2	4



Complete the table for each equation. Then, draw the graph.

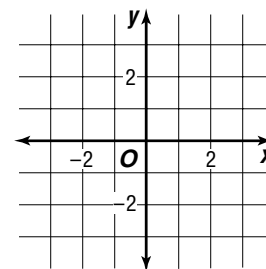
1. $y = |x|$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



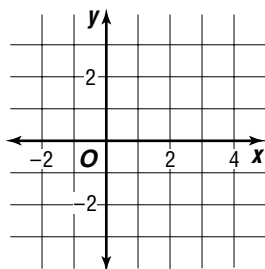
2. $y = |x| - 2$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



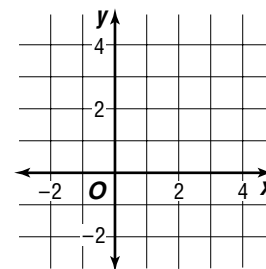
3. $y = |x - 1|$

x	y
-2	
-1	
0	
1	
2	
3	
4	



4. $y = |2 - x|$

x	y
-2	
-1	
0	
1	
2	
3	
4	



Study Guide

Graphing Inequalities In Two Variables

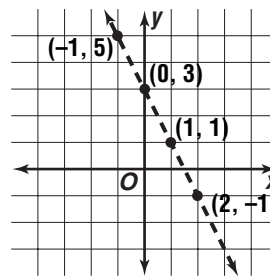
Inequalities, like equations, may have two variables instead of one. The solution of an inequality having two variables contains many ordered pairs. The graph of these ordered pairs fills an area of the coordinate plane called a **half-plane**. The graph of the related equation defines the **boundary** or edge for each half-plane.

Example: Graph $y < -2x + 3$.

Step 1 Determine the boundary by graphing the related equation, $y = -2x + 3$.

Make a table of values.

x	$-2x + 3$	y
-2	$-2(-2) + 3$	7
-1	$-2(-1) + 3$	5
0	$-2(0) + 3$	3
1	$-2(1) + 3$	1
2	$-2(2) + 3$	-1



Step 2 Draw a dashed line because the boundary is not included.
Note: If the inequality involved \leq or \geq , the boundary would be included, and you would make the boundary a solid line.

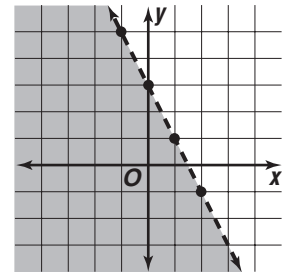
Step 3 Use a point not on the boundary to find which half-plane is the solution. Use $(0, 0)$.

$$y < -2x + 3$$

$$0 \stackrel{?}{<} -2(0) + 3$$

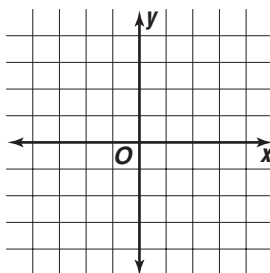
$$0 < 3 \text{ true}$$

Since $0 < 3$ is true, shade the half-plane containing $(0, 0)$. *Note:* If the result were false, you would shade the other half-plane.

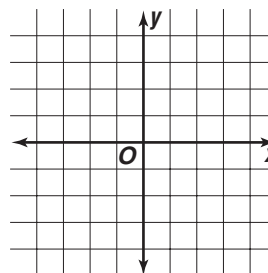


Graph each inequality.

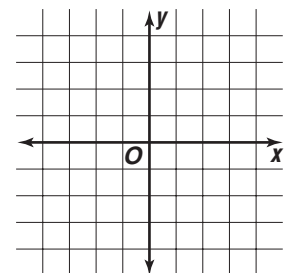
1. $y > x + 2$



2. $y \leq x - 2$



3. $y \leq -2x + 2$

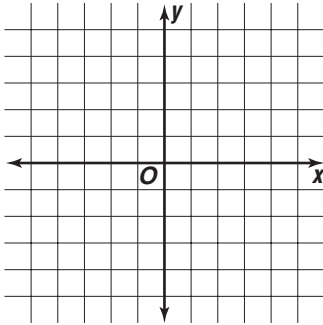


Practice

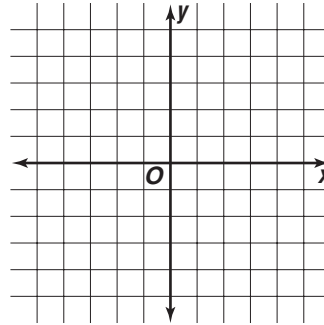
Graphing Inequalities in Two Variables

Graph each inequality.

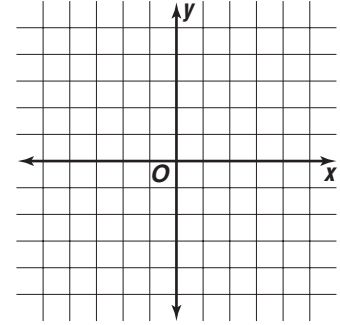
1. $y > -2$



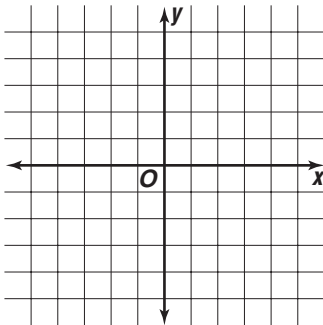
2. $y \leq x + 3$



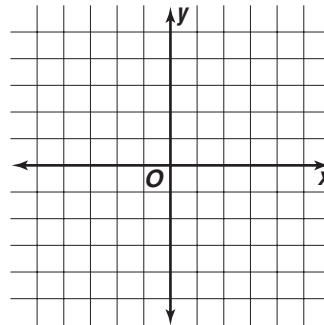
3. $y \geq -x + 1$



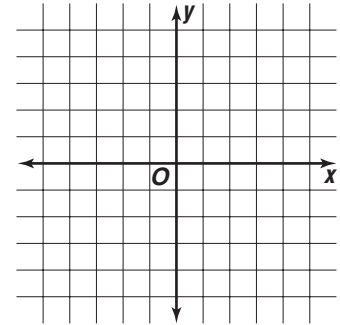
4. $y < 3x + 3$



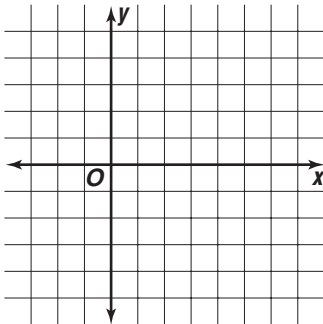
5. $x + y \leq -4$



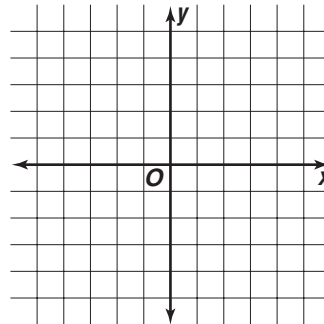
6. $2x + y > 2$



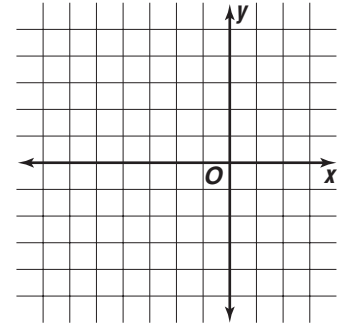
7. $2x - y \geq 10$



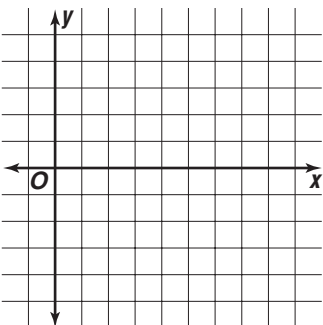
8. $-3x + y > 9$



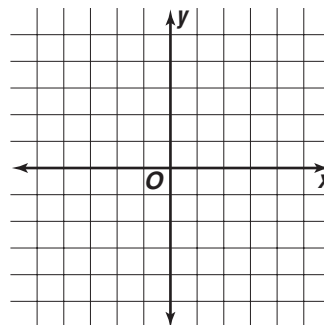
9. $x + 2y \leq -6$



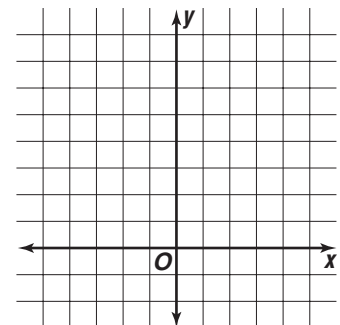
10. $x - 4y < 8$



11. $2x + 2y \geq 6$



12. $-4x + 2y \leq 12$



Enrichment

Inequalities with Triangles

Recall that a line segment can be named by the letters of its endpoints. Line segment AB (written as " \overline{AB} ") has points A and B for endpoints. The *length* of AB is written without the bar as AB .

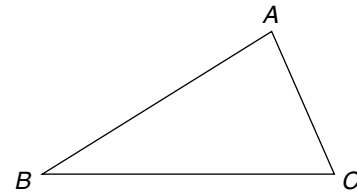
$$AB < BC$$

$$\angle A < \angle B$$

The statement on the left above shows that \overline{AB} is shorter than \overline{BC} . The statement on the right above shows that the measure of angle A is less than that of angle B .

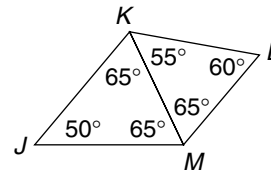
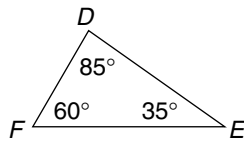
These three inequalities are true for any triangle ABC , no matter how long the sides are.

- $AB + BC > AC$
- If $AB > AC$, then $\angle C > \angle B$.
- If $\angle C > \angle B$, then $AB > AC$.



Use the three triangle inequalities for these problems.

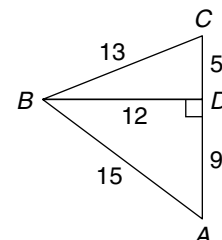
- List the sides of triangle DEF in order of increasing length.
- In the figure below, which line segment is the shortest?



- Explain why the lengths 5 cm, 10 cm, and 20 cm could not be used to make a triangle.
- Two sides of a triangle measure 3 in. and 7 in. Between which two values must the third side be?

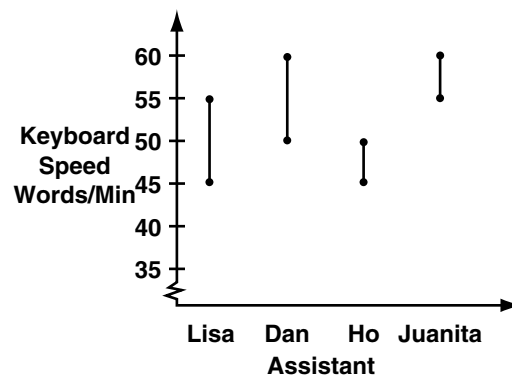
- In triangle XYZ , $XY = 15$, $YZ = 12$, and $XZ = 9$. Which is the greatest angle? Which is the least?

- List the angles $\angle A$, $\angle C$, $\angle ABC$, and $\angle ABD$, in order of increasing size.



Keyboarding Times (Administrative Assistant)

The efficiency of an administrative assistant depends in part on the assistant's keyboarding speed, measured in words per minute. The chart at the right shows the efficiency ratings for four administrative assistants. Lisa, for example, can keyboard between 45 words per minute and 55 words per minute.



Suppose a manager gives Lisa a document containing 1200 words. How long will it take Lisa to keyboard the document?

Let x represent the number of minutes it will take to do the keyboarding.

Since 45 words per minute is a lower estimate and 55 words per minute is an upper estimate, solve the inequalities $45x \leq 1200$ and $55x \geq 1200$.

$$45x \leq 1200$$

$$55x \geq 1200$$

$$\frac{45x}{45} \leq \frac{1200}{45}$$

$$\frac{55x}{55} \geq \frac{1200}{55}$$

$$x \leq 26.667$$

$$x \geq 21.818$$

It will take Lisa between 21 minutes and 27 minutes to keyboard the document.

Solve.

- Suppose a manager gives Ho a document containing 2800 words. How long will it take Ho to keyboard the document?
- Suppose a manager gives Juanita a document containing 18,000 words. How many hours will it take her to enter the document on a computer?
- Which administrative assistant should be given a 12,000-word document for keyboarding if the manager wants it entered in the least amount of time? Explain.