

Graphing Systems of Equations

The ordered pair $(-1, -3)$ is the solution of the system of equations

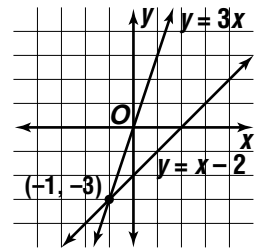
$$\begin{aligned} y &= x - 2 \\ y &= 3x \end{aligned}$$

because when -1 is substituted for x and -3 is substituted for y , both equations are true.

$$\begin{array}{ll} y = x - 2 & y = 3x \\ -3 \stackrel{?}{=} -1 - 2 & -3 \stackrel{?}{=} 3(-1) \\ -3 = -3 \checkmark & -3 = -3 \checkmark \end{array}$$

You can also graph both equations to show that $(-1, -3)$ is the solution of the system.

The graphs appear to intersect at $(-1, -3)$. Since $(-1, -3)$ is the solution of each equation, it is the solution of the system of equations.



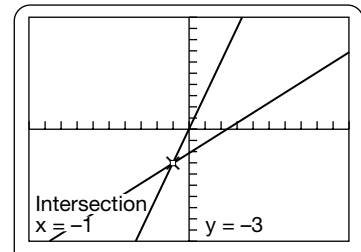
You can also use a graphing calculator to solve the system of equations.

Step 1 Enter these keystrokes in the Y= screen:

X, T, θ, n $-$ 2 **ENTER**
3 X, T, θ, n **ENTER** **GRAPH**

Step 2 Use the INTERSECT feature to find the intersection point.

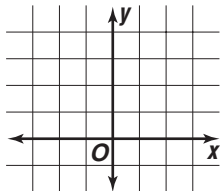
2nd **[CALC]** 5 **ENTER** **ENTER** **ENTER**



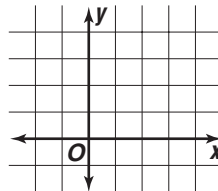
The solution is $(-1, -3)$.

Solve each system of equations by graphing.

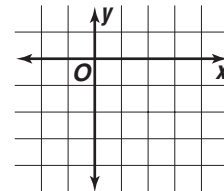
1. $x = -1$
 $y = 3$



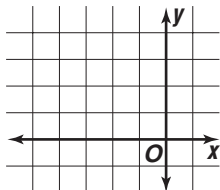
2. $y = 2$
 $y = x$



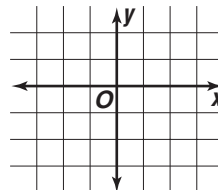
3. $y = -3$
 $x = 2$



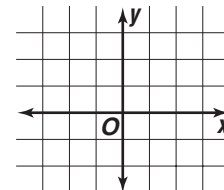
4. $y = -3x$
 $y = x + 4$



5. $y = 1 - x$
 $y = 2x - 5$



6. $y = -x - 2$
 $y = 3x + 2$

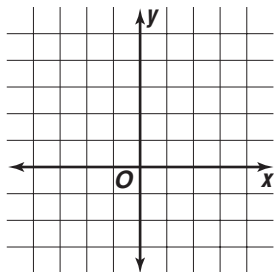


Practice

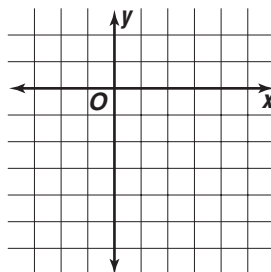
Graphing Systems of Equations

Solve each system of equations by graphing.

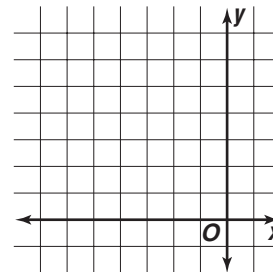
$$\begin{aligned} 1. \quad & y = 3x \\ & y = -x + 4 \end{aligned}$$



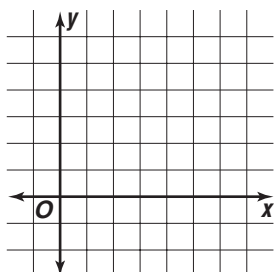
$$\begin{aligned} 2. \quad & y = x - 4 \\ & y = 2x - 3 \end{aligned}$$



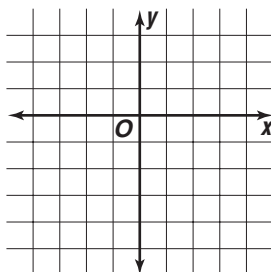
$$\begin{aligned} 3. \quad & x = -3 \\ & y = x + 6 \end{aligned}$$



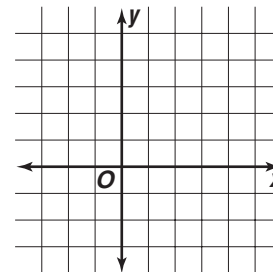
$$\begin{aligned} 4. \quad & x - y = 1 \\ & y = 5 \end{aligned}$$



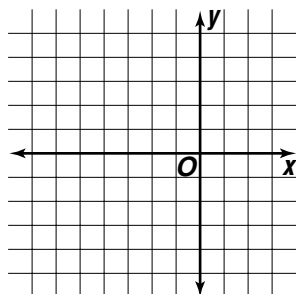
$$\begin{aligned} 5. \quad & x + y = -1 \\ & x - y = 3 \end{aligned}$$



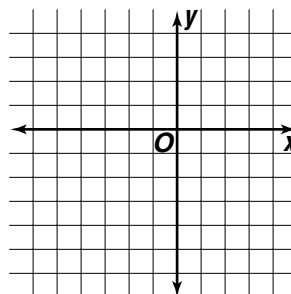
$$\begin{aligned} 6. \quad & x + y = 2 \\ & y = -2x + 4 \end{aligned}$$



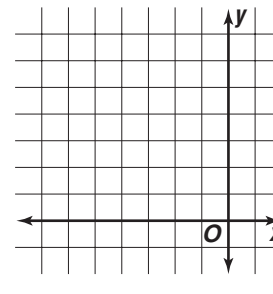
$$\begin{aligned} 7. \quad & y = x + 3 \\ & y = -x - 5 \end{aligned}$$



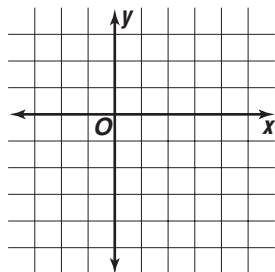
$$\begin{aligned} 8. \quad & -x + y = 2 \\ & -2x + y = 7 \end{aligned}$$



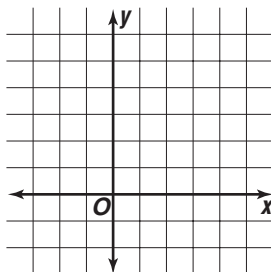
$$\begin{aligned} 9. \quad & y = x + 6 \\ & y = 2 \end{aligned}$$



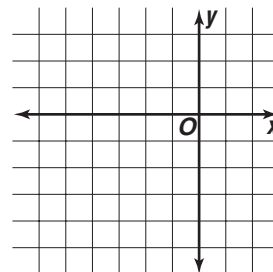
$$\begin{aligned} 10. \quad & x - y = 4 \\ & y = -2x + 2 \end{aligned}$$



$$\begin{aligned} 11. \quad & y = x + 2 \\ & 3x + y = 10 \end{aligned}$$



$$\begin{aligned} 12. \quad & y = x + 2 \\ & 2x + y = -1 \end{aligned}$$



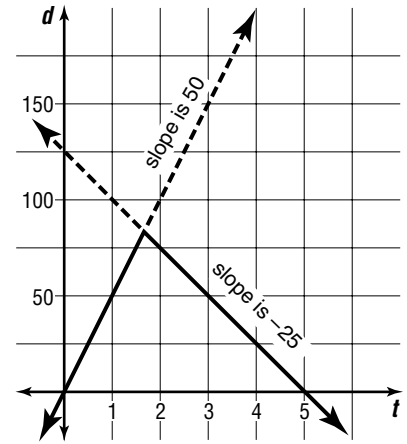
Enrichment

Graphing a Trip

The formula $d = rt$ is used to solve many types of problems. If you graph an equation such as $d = 50t$, the graph is a model for a car going at 50 mi/h. The time the car travels is t ; the distance in miles the car covers is d . The slope of the line is the speed.

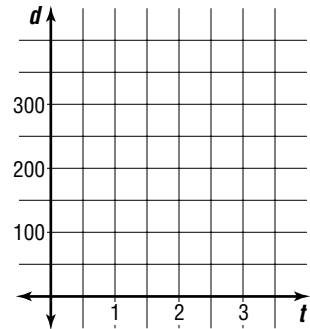
Suppose you drive to a nearby town and return. You average 50 mi/h on the trip out but only 25 mi/h on the trip home. The round trip takes 5 hours. How far away is the town?

The graph at the right represents your trip. Notice that the return trip is shown with a negative slope because you are driving in the opposite direction.

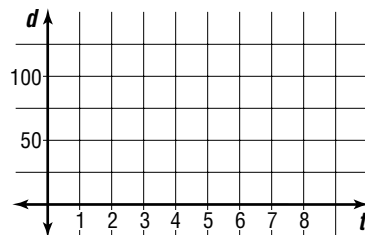
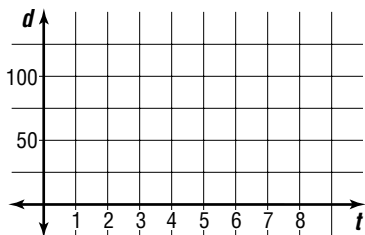


Solve each problem.

1. Estimate the answer to the problem in the above example. About how far away is the town?
2. Graph this trip and solve the problem. An airplane has enough fuel for 3 hours of safe flying. On the trip out the pilot averages 200 mi/h flying against a headwind. On the trip back, the pilot averages 250 mi/h. How long a trip out can the pilot make?
3. Graph this trip and solve the problem. You drive to a town 100 miles away. On the trip out you average 25 mi/h. On the trip back you average 50 mi/h. How many hours do you spend driving?



4. Graph this trip and solve the problem. You drive at an average speed of 50 mi/h to a discount shopping plaza, spend 2 hours shopping, and then return at an average speed of 25 mi/h. The entire trip takes 8 hours. How far away is the shopping plaza?

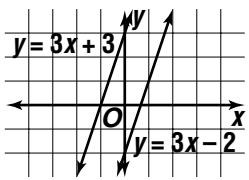
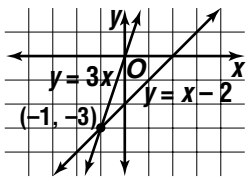
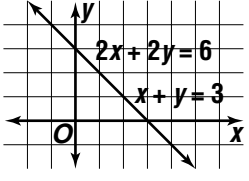


Study Guide

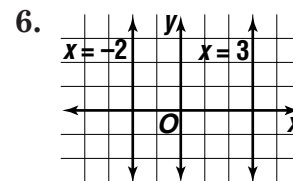
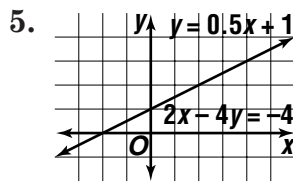
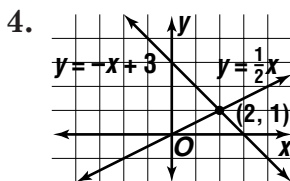
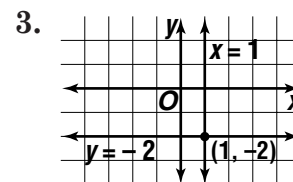
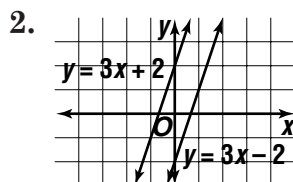
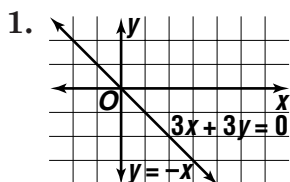
Solutions of Systems of Equations

A system of linear equations is:

- **inconsistent** if the graphs of the equations are parallel. An inconsistent system has no solution.
- **consistent and independent** if the graphs of the equations intersect at one point. A consistent and independent system has one solution.
- **consistent and dependent** if the equations have the same graph. A consistent and dependent system has infinitely many solutions.

Graph	Description of Graph	Number of Solutions	Type of System
	parallel lines	0	inconsistent
	intersecting lines	1	consistent and independent
	same line	infinitely many	consistent and dependent

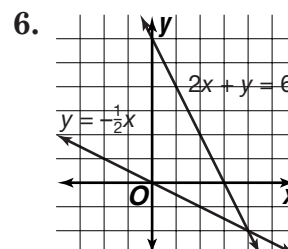
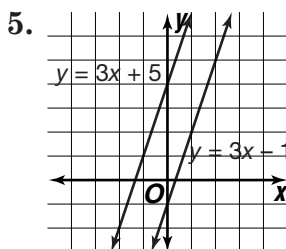
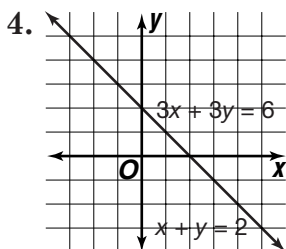
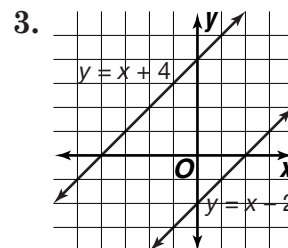
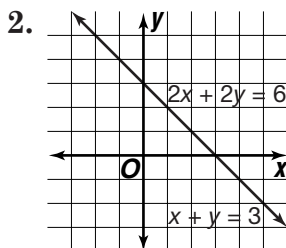
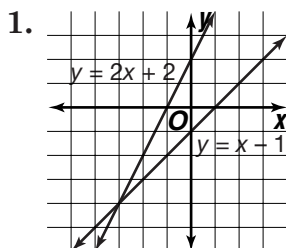
State whether each system is consistent and independent, consistent and dependent, or inconsistent.



Practice

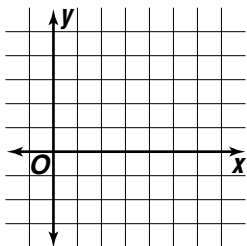
Solutions of Systems of Equations

State whether each system is consistent and independent, consistent and dependent, or inconsistent.

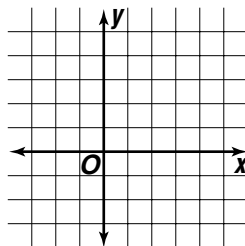


Determine whether each system of equations has one solution, no solution, or infinitely many solutions by graphing. If the system has one solution, name it.

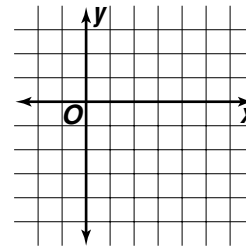
7. $2x + y = 4$
 $4x + 2y = 8$



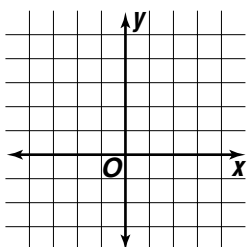
8. $y = x - 1$
 $x + y = 3$



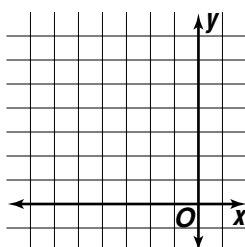
9. $y = x - 2$
 $y = x - 5$



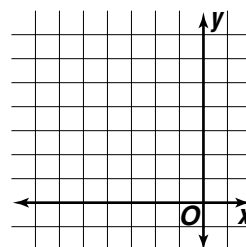
10. $y = 2x$
 $y = 2x + 3$



11. $y = x + 5$
 $-x + y = 5$

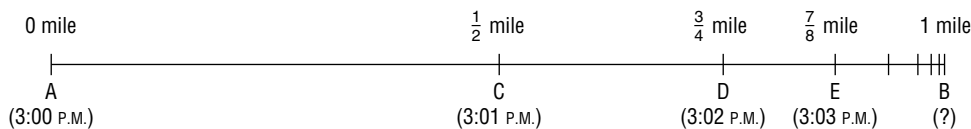


12. $x - y = -5$
 $y = 2x + 6$



Convergence, Divergence, and Limits

Imagine that a runner runs a mile from point A to point B. But, this is not an ordinary race! In the first minute, he runs one-half mile, reaching point C. In the next minute, he covers one-half the remaining distance, or $\frac{1}{4}$ mile, reaching point D. In the next minute he covers one-half the remaining distance, or $\frac{1}{8}$ mile, reaching point E.



In this strange race, the runner approaches closer and closer to point B, but never gets there. However close he is to B, there is still some distance remaining, and in the next minute he can cover only half of that distance.

This race can be modeled by the infinite sequence

$$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \dots$$

The terms of the sequence get closer and closer to 1. An infinite sequence that gets arbitrarily close to some number is said to **converge** to that number. The number is the limit of the sequence.

Not all infinite sequences converge. Those that do not are called **divergent**.

Write C if the sequence converges and D if it diverges. If the sequence converges, make a reasonable guess for its limit.

1. 2, 4, 6, 8, 10, ...

2. 0, 3, 0, 3, 0, 3, ...

3. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

4. 0.9, 0.99, 0.999, 0.9999, ...

5. -5, 5, -5, 5, -5, 5, ...

6. 0.1, 0.2, 0.3, 0.4, ...

7. $2\frac{1}{4}, 2\frac{3}{4}, 2\frac{7}{8}, 2\frac{15}{16}, \dots$

8. $6, 5\frac{1}{2}, 5\frac{1}{3}, 5\frac{1}{4}, 5\frac{1}{5}, \dots$

9. 1, 4, 9, 16, 25, ...

10. $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots$

11. Create one convergent sequence and one divergent sequence. Give the limit for your convergent sequence.

Study Guide

Substitution

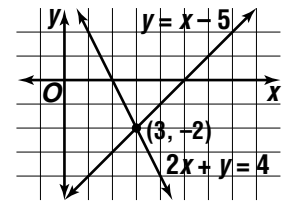
You can use a method called *substitution* to solve a system of linear equations.

Example: Use substitution to solve the system of equations.

$$\begin{aligned}y &= x - 5 \\2x + y &= 4\end{aligned}$$

The first equation tells you that y is equal to $x - 5$, so substitute $x - 5$ for y in the second equation. Then solve for x .

$$\begin{aligned}2x + y &= 4 \\2x + x - 5 &= 4 && \text{Replace } y \text{ with } x - 5. \\3x - 5 &= 4 \\3x - 5 + 5 &= 4 + 5 && \text{Add 5 to each side.} \\3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} && \text{Divide each side by 3.} \\x &= 3\end{aligned}$$



Now choose one of the original equations. Substitute 3 for x in the equation you have chosen. Then solve for y .

$$\begin{aligned}y &= x - 5 && \text{Choose one of the original equations.} \\y &= 3 - 5 && \text{Substitute 3 for } x. \\y &= -2\end{aligned}$$

The solution of the system of equations is $(3, -2)$.

Use substitution to solve each system of equations.

1. $y = x$
 $x + y = 6$

2. $x + y = 0$
 $y = 3$

3. $y = -3x$
 $x + y = 8$

4. $y = x - 3$
 $x + 2y = 6$

5. $x + y = 0$
 $x - y = 4$

6. $y = 2x + 3$
 $y - x = 10$

7. $x = -1$
 $x + y = 5$

8. $y = 3 - 2x$
 $4x + y = 5$

9. $x = y - 10$
 $3x = y$

10. $2y = x + 6$
 $y = 2x + 3$

11. $3x = y + 5$
 $y = 2x - 5$

12. $y = 4x - 6$
 $y = x - 3$

13. $x = 5y - 12$
 $x - y = 0$

14. $3y = 2x - 3$
 $y = -\frac{1}{3}x + 2$

15. $y = x - 6$
 $5x - y = 6$

Substitution

Use substitution to solve each system of equations.

1. $y = x + 8$
 $x + y = 2$

2. $y = 2x$
 $5x - y = 9$

3. $y = x + 2$
 $3x + 3y = 6$

4. $x = 3y$
 $2x + 4y = 10$

5. $x = y + 9$
 $x + y = -7$

6. $y = 2x + 1$
 $2x - y = 3$

7. $x = 3y$
 $2x + 3y = 15$

8. $x - 2y = 4$
 $3x = 6y + 12$

9. $x = 5y - 2$
 $2x + 2y = 4$

10. $4y + 2x = 24$
 $x = 3y + 2$

11. $y = 3x + 8$
 $4x + 2y = 6$

12. $x = 3y + 10$
 $2x + 2y = -12$

13. $x + 2y = -4$
 $-2x - 3y = 9$

14. $5x + 2y = 7$
 $4x + y = 8$

15. $x = 2y + 11$
 $3x + 2y = 9$

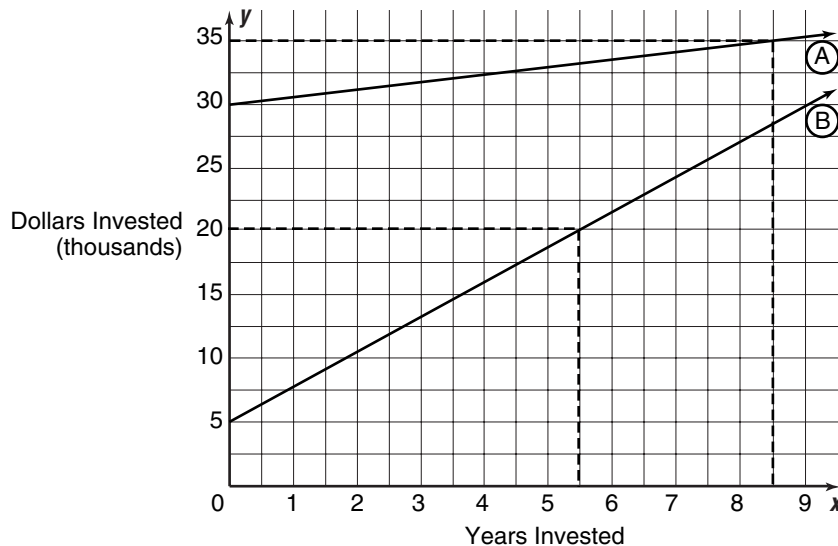
16. $x - 2y = -7$
 $5x - 7y = -8$

17. $6x - 4y = -5$
 $2x + y = 3$

18. $x + 3y = 10$
 $4x - 5y = 6$

Enrichment**Investments**

The graph below represents two different investments. Line *A* represents an initial investment of \$30,000 at a bank paying passbook-savings interest. Line *B* represents an initial investment of \$5000 in a profitable mutual fund with dividends reinvested and capital gains accepted in shares. By deriving the equation, $y = mx + b$, for *A* and *B*, a projection of the future can be made.

**Solve.**

- The y -intercept, b , is the initial investment. Find b for each of the following.
 - line *A*
 - line *B*
- The slope of the line, m , is the rate of return. Find m for each of the following.
 - line *A*
 - line *B*
- What are the equations of each of the following lines?
 - line *A*
 - line *B*

Answer each of the following, assuming that the growth of each investment continues in the same pattern.

- What will be value of the mutual fund after the 11th year?
- What will be the value of the bank account after the 11th year?
- When will the mutual fund and the bank account be of equal value?
- In the long term, which investment has the greater payoff?

Study Guide

Elimination Using Addition and Subtraction

In Lesson 13–3 you used substitution to solve systems of linear equations. You can also use the *elimination method* to solve systems of linear equations. When you use the elimination method, you eliminate one of the variables by adding or subtracting the equations. Add the equations to eliminate the variable whose coefficients are additive inverses. Subtract the equations to eliminate the variable whose coefficients are the same.

Example: Use elimination to solve the system of equations.

$$2x - y = -3$$

$$2x + y = -9$$

Step 1 The coefficients of y are -1 and 1 , so add the equations to eliminate the y terms. Then solve for x .

$$\begin{array}{r} 2x - y = -3 \\ (+)2x + y = -9 \\ \hline \end{array} \text{Add the equations.}$$

$$4x + 0 = -12 \text{ The } y \text{ terms are eliminated.}$$

$$4x = -12 \text{ Divide each side by 4.}$$

$$x = -3 \text{ The value of } x \text{ is } -3.$$

Step 2 Replace x in one of the original equations with -3 . Then solve for y .

$$2x + y = -9 \text{ Choose an equation.}$$

$$2(-3) + y = -9 \text{ Replace } x \text{ with } -3.$$

$$-6 + y = -9$$

$$-6 + y + 6 = -9 + 6 \text{ Add 6 to each side.}$$

$$y = -3 \text{ The value of } y \text{ is } -3.$$

The solution of the system of equations is $(-3, -3)$.

You could also use subtraction to eliminate the x terms in the example.

Step 1 The coefficients of x are both 2 , so subtract to eliminate the x terms.

$$\begin{array}{r} 2x - y = -3 \\ (-)2x + y = -9 \\ \hline \end{array} \text{Subtract the equations.}$$

$$0 - 2y = 6 \text{ The } x \text{ terms are eliminated.}$$

$$-2y = 6$$

$$y = -3$$

Step 2 Solve for x .

$$2x + y = -9$$

$$2x + (-3) = -9 \text{ Replace } y \text{ with } -3.$$

$$2x = -6$$

$$x = -3$$

The solution is $(-3, -3)$.

Use elimination to solve the system of equations.

1. $x - y = 2$
 $x + y = 0$

2. $3x + 2y = 6$
 $-3x + y = 0$

3. $2x - y = -4$
 $-3x - y = 6$

4. $2x + y = 6$
 $3x + y = 5$

5. $3x - 4y = 11$
 $3x + 5y = -7$

6. $x + y = 6$
 $-2x + y = -3$

Practice***Elimination Using Addition and Subtraction***

Use elimination to solve each system of equations.

1. $x + y = 4$
 $x - y = -6$

2. $x - y = 7$
 $x + y = 1$

3. $3x + y = 12$
 $x + y = 8$

4. $x + 5y = -12$
 $x + 2y = -9$

5. $x + 2y = 9$
 $3x - 2y = 3$

6. $4x + 2y = 2$
 $-4x - 3y = 3$

7. $4x - 3y = 10$
 $2x - 3y = 2$

8. $2x + 5y = 1$
 $2x + 10y = 10$

9. $3y = x + 4$
 $2x + 3y = 19$

10. $2x = y - 4$
 $2x + 6y = 3$

11. $4y = 2x + 8$
 $5x - 4y = 22$

12. $2x + y = 6$
 $2x - 2y = -12$

13. $-3x - y = 24$
 $3x - 2y = 3$

14. $2x + 3y = 8$
 $y = 2x + 8$

15. $-7x = y - 4$
 $5x - y = 8$

16. $3x + 5y = 7$
 $4x + 5y = 1$

17. $6x - 3y = 3$
 $6x - 5y = -3$

18. $y = 2x + 4$
 $2x - 4y = 8$

Arithmetic Series

An **arithmetic series** is a series in which each term after the first may be found by adding the same number to the preceding term. Let S stand for the following series in which each term is 3 more than the preceding one.

$$S = 2 + 5 + 8 + 11 + 14 + 17 + 20$$

The series remains the same if we reverse the order of all the terms. So let us reverse the order of the terms and add one series to the other, term by term. This is shown at the right.

$$\begin{aligned} S &= 2 + 5 + 8 + 11 + 14 + 17 + 20 \\ S &= 20 + 17 + 14 + 11 + 8 + 5 + 2 \\ 2S &= 22 + 22 + 22 + 22 + 22 + 22 + 22 \\ 2S &= 7(22) \\ S &= \frac{7(22)}{2} = 7(11) = 77 \end{aligned}$$

Let a represent the first term of the series.

Let l represent the last term of the series.

Let n represent the number of terms in the series.

In the preceding example, $a = 2$, $l = 20$, and $n = 7$. Notice that when you add the two series, term by term, the sum of each pair of terms is 22. That sum can be found by adding the first and last terms, $2 + 20$ or $a + l$. Notice also that there are 7, or n , such sums. Therefore, the value of $2S$ is $7(22)$, or $n(a + l)$ in the general case. Since this is twice the sum of the series, you can use the following formula to find the sum of any arithmetic series.

$$S = \frac{n(a + l)}{2}$$

Example 1: Find the sum: $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$

$$a = 1, l = 9, n = 9, \text{ so } S = \frac{9(1 + 9)}{2} = \frac{9 \cdot 10}{2} = 45$$

Example 2: Find the sum: $-9 + (-5) + (-1) + 3 + 7 + 11 + 15$

$$a = -9, l = 15, n = 7, \text{ so } S = \frac{7(-9 + 15)}{2} = \frac{7 \cdot 6}{2} = 21$$

Find the sum of each arithmetic series.

- $3 + 6 + 9 + 12 + 15 + 18 + 21 + 24$
- $10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50$
- $-21 + (-16) + (-11) + (-6) + (-1) + 4 + 9 + 14$
- even whole numbers from 2 through 100
- odd whole numbers between 0 and 100

Study Guide

Elimination Using Multiplication

In some systems of equations, adding or subtracting the equations will not eliminate one of the variables. When this is true, you can eliminate by first multiplying one or both of the equations by a number, and then adding or subtracting the equations.

Example: Use elimination to solve the system of equations.

$$\begin{aligned} 2x + y &= 6 \\ 3x + 3y &= 9 \end{aligned}$$

Adding or subtracting the equations will not eliminate the x terms or the y terms. If you multiply the first equation by -3 , however, the y terms will be additive inverses. Then you can add the equations to eliminate the y terms.

$$\begin{array}{r} 2x + y = 6 \\ 3x + 3y = 9 \end{array} \quad \begin{array}{c} \text{Multiply by } -3. \\ \rightarrow \end{array} \quad \begin{array}{r} -3(2x + y = 6) \rightarrow -6x - 3y = -18 \\ 3x + 3y = 9 \quad (+) 3x + 3y = 9 \\ \hline -3x + 0 = -9 \\ -3x = -9 \\ x = 3 \end{array}$$

Now find the value of y .

$$\begin{aligned} 2x + y &= 6 && \text{Choose either of the original equations.} \\ 2(3) + y &= 6 && \text{Replace } x \text{ with } 3. \\ 6 + y &= 6 && \text{Simplify. Then subtract 6 from each side.} \\ y &= 0 \end{aligned}$$

Check:

$$\begin{array}{r} 2x + y = 6 \\ 2(3) + 0 \stackrel{?}{=} 6 \\ 6 = 6 \checkmark \end{array} \quad \begin{array}{r} 3x + 3y = 9 \\ 3(3) + 3(0) \stackrel{?}{=} 9 \\ 9 = 9 \checkmark \end{array}$$

The solution of this system of equations is $(3, 0)$.

Use elimination to solve each system of equations.

1. $x + 2y = 1$
 $3x + y = 8$

2. $x + 11y = -6$
 $2x + y = 9$

3. $8x - 3y = -32$
 $x - y = 1$

4. $3x - 5y = 8$
 $x + 2y = -1$

5. $3x - 4y = 5$
 $x + 7y = 10$

6. $2x - y = 2$
 $3x - 2y = 3$

7. $3x + 5y = 9$
 $9x + 2y = -12$

8. $8x + 9y = -45$
 $x + 6y = 9$

9. $12x - 10y = 30$
 $2x + 5y = 15$

Practice***Elimination Using Multiplication******Use elimination to solve each system of equations.***

1.
$$\begin{aligned}x + 3y &= 6 \\ 2x - 7y &= -1\end{aligned}$$

2.
$$\begin{aligned}9x + 3y &= 12 \\ 2x + y &= 5\end{aligned}$$

3.
$$\begin{aligned}3x - y &= 14 \\ 5x + 4y &= 12\end{aligned}$$

4.
$$\begin{aligned}3x - 3y &= -3 \\ 2x - y &= -5\end{aligned}$$

5.
$$\begin{aligned}3x + y &= 2 \\ 6x + 2y &= 4\end{aligned}$$

6.
$$\begin{aligned}5x - y &= 16 \\ -4x - 3y &= 10\end{aligned}$$

7.
$$\begin{aligned}5x + 2y &= 24 \\ 10x - 5y &= -15\end{aligned}$$

8.
$$\begin{aligned}3x + 4y &= 6 \\ 7x + 8y &= 10\end{aligned}$$

9.
$$\begin{aligned}2x - 3y &= 5 \\ 3x + 9y &= 21\end{aligned}$$

10.
$$\begin{aligned}3x + 2y &= 11 \\ 6x + 3y &= 13\end{aligned}$$

11.
$$\begin{aligned}6x - 2y &= 4 \\ 2x - 5y &= -3\end{aligned}$$

12.
$$\begin{aligned}-7x - 3y &= -5 \\ 5x + 6y &= 19\end{aligned}$$

13.
$$\begin{aligned}5x - 10y &= -3 \\ -3x - 5y &= 15\end{aligned}$$

14.
$$\begin{aligned}2x + 3y &= 2 \\ 6x + 9y &= 5\end{aligned}$$

15.
$$\begin{aligned}2x + 4y &= 6 \\ 3x + 6y &= 12\end{aligned}$$

16.
$$\begin{aligned}3x + 3y &= 9 \\ 5x + 4y &= 10\end{aligned}$$

17.
$$\begin{aligned}2x - 7y &= 5 \\ 3x - 6y &= 12\end{aligned}$$

18.
$$\begin{aligned}2x - 4y &= 18 \\ -5x - 6y &= 3\end{aligned}$$

Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

For example, here is how to approximate an equation of the parabola through the points $(0, -2)$, $(3, 0)$, and $(5, 2)$.

Use the general equation $y = ax^2 + bx + c$. By substituting the given values for x and y , you get three equations.

$$\begin{aligned}(0, -2): & -2 = c \\(3, 0): & 0 = 9a + 3b + c \\(5, 2): & 2 = 25a + 5b + c\end{aligned}$$

First, substitute -2 for c in the second and third equations. Then solve those two equations as you would any system of two equations. Multiply the second equation by 5 and the third equation by -3 .

$$\begin{aligned}0 &= 45a + 15b - 10 \\-6 &= -75a - 15b + 6 \\-6 &= -30a - 15b - 4 \\a &= \frac{1}{15}\end{aligned}$$

To find b , substitute $\frac{1}{15}$ for a in either the second or third equation.

$$\begin{aligned}0 &= 9\left(\frac{1}{15}\right) + 3b - 2 \\b &= \frac{7}{15}\end{aligned}$$

The equation of a parabola through the three points is

$$y = \frac{1}{15}x^2 + \frac{7}{15}x - 2.$$

Find the equation of a parabola through each set of three points.

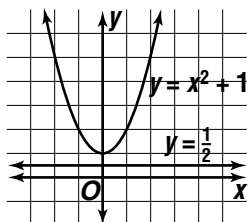
1. $(1, 5)$, $(0, 6)$, $(2, 3)$
2. $(-5, 0)$, $(0, 0)$, $(8, 100)$
3. $(4, -4)$, $(0, 1)$, $(3, -2)$
4. $(1, 3)$, $(6, 0)$, $(0, 0)$
5. $(2, 2)$, $(5, -3)$, $(0, -1)$
6. $(0, 4)$, $(4, 0)$, $(-4, 4)$

Study Guide

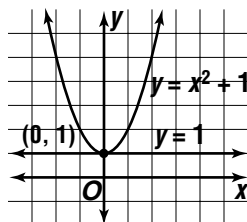
Solving Quadratic-Linear Systems of Equations

A system of equations that contains both a quadratic equation and a linear equation is called a **quadratic-linear** system of equations.

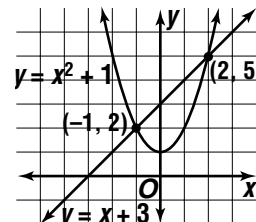
One way to solve quadratic-linear systems is to graph them and identify any ordered pairs that satisfy both equations. Another way is to use the substitution method.



The graphs do not intersect, so the system has no solution.



The graphs intersect at one point, so the system has one solution.



The graphs intersect at two points, so the system has two solutions.

Example: Use substitution to solve the system of equations.

$$y = -2$$

$$y = x^2 - 11$$

Since $y = -2$, substitute -2 for y in the second equation. Then solve for x .

$$y = x^2 - 11$$

$$-2 = x^2 - 11 \quad \text{Replace } y \text{ with } -2.$$

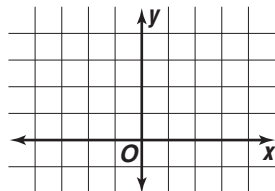
$$9 = x^2 \quad \text{Add 11 to each side.}$$

$$3 = x \text{ or } -3 = x \quad \text{Take the square root of each side.}$$

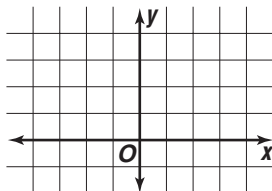
The solutions of the system of equations are $(-3, -2)$ and $(3, -2)$. Graphing the equations will show that they intersect at $(-3, -2)$ and $(3, -2)$.

Solve each system of equations by graphing.

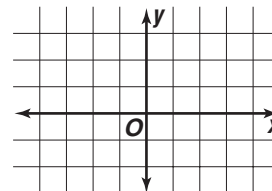
1. $x = -1$
 $y = 2x^2$



2. $y = -1$
 $y = x^2$



3. $y = x$
 $y = x^2 - 2$



Use substitution to solve each system of equations.

4. $y = 4$
 $y = x^2$

5. $x = -1$
 $y = x^2$

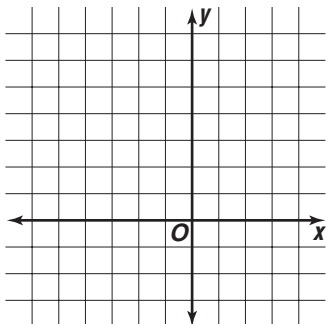
6. $y = 5$
 $y = x^2 - 4$

Practice

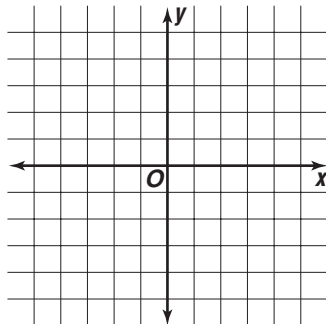
Solving Quadratic-Linear Systems of Equations

Solve each system of equations by graphing.

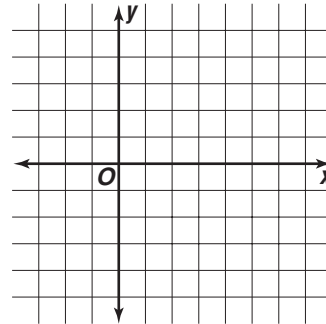
$$\begin{aligned} 1. \quad & y = x^2 + 2 \\ & y = x + 4 \end{aligned}$$



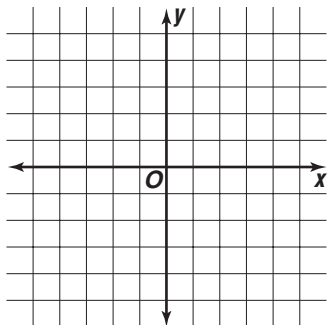
$$\begin{aligned} 2. \quad & y = x^2 - 1 \\ & y = x - 2 \end{aligned}$$



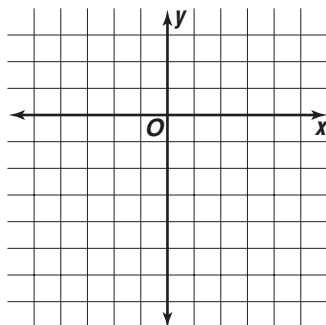
$$\begin{aligned} 3. \quad & y = -x^2 + 3 \\ & y = 3 \end{aligned}$$



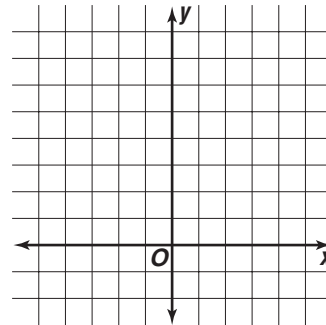
$$\begin{aligned} 4. \quad & y = x^2 + 1 \\ & y = -x - 1 \end{aligned}$$



$$\begin{aligned} 5. \quad & y = -x^2 \\ & y = -2x + 1 \end{aligned}$$



$$\begin{aligned} 6. \quad & y = x^2 - 2 \\ & y = x + 4 \end{aligned}$$



Use substitution to solve each system of equations.

$$\begin{aligned} 7. \quad & y = -x^2 + 1 \\ & y = x - 1 \end{aligned}$$

$$\begin{aligned} 8. \quad & y = x^2 + 2 \\ & y = -4 \end{aligned}$$

$$\begin{aligned} 9. \quad & y = x^2 - 5 \\ & x = -3 \end{aligned}$$

$$\begin{aligned} 10. \quad & y = -6x^2 + 1 \\ & y = x + 1 \end{aligned}$$

$$\begin{aligned} 11. \quad & y = 2x^2 + 3 \\ & y = x + 2 \end{aligned}$$

$$\begin{aligned} 12. \quad & y = x^2 + x - 4 \\ & y = x - 3 \end{aligned}$$

Study Guide

Graphing Systems of Inequalities

To graph a system of linear inequalities, first graph a boundary line for each inequality. Then shade on one side of each boundary line. The region where the shaded areas overlap contains the solutions of the system of inequalities.

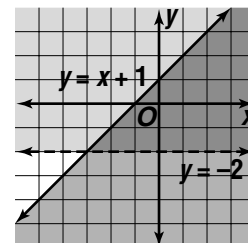
Example: Solve the system of inequalities by graphing.

$$y > -2$$

$$y \leq x + 1$$

Step 1 Graph the boundary lines $y = -2$ and $y = x + 1$. Since y is *greater than* -2 , make the line $y = -2$ dashed. The line $y = x + 1$ is solid because y is *less than or equal to* $x + 1$.

Step 2 Because $y > -2$ has a *greater than* symbol, shade above the boundary line. Shade below the boundary line for $y \leq x + 1$ because this inequality contains a *less than* symbol.



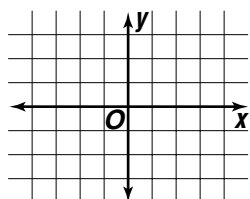
The region where the shaded areas intersect is the solution of the system of inequalities.

Use the following rules to help you graph inequalities.

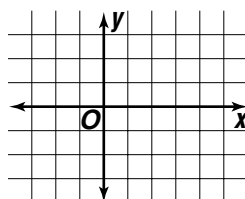
Inequality	Example	Boundary Line	Where to Shade
$y < ax + b$	$y < 2x$	dashed	below the line
$y \leq ax + b$	$y \leq -x + 1$	solid	below the line
$y > ax + b$	$y > 5x - 2$	dashed	above the line
$y \geq ax + b$	$y \geq -4x$	solid	above the line

Solve each system of inequalities by graphing. If the system does not have a solution, write no solution.

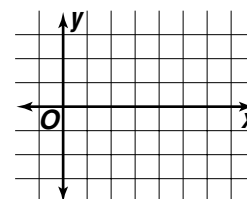
1. $y < 1$
 $y > -3$



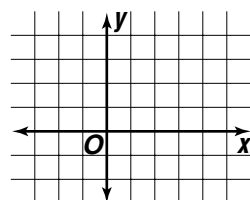
2. $x > 4$
 $x < -1$



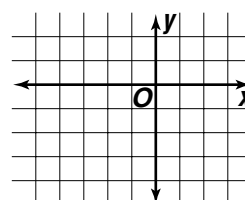
3. $y \geq -2$
 $x \leq 5$



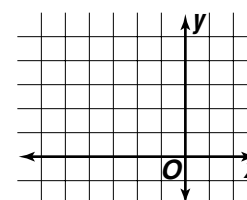
4. $x \leq 2$
 $y > x$



5. $y < x$
 $x \leq -1$



6. $y \geq 0$
 $y \leq x + 3$

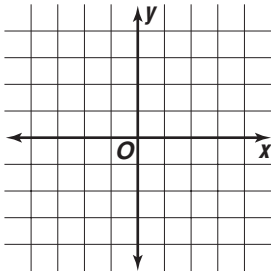


Practice

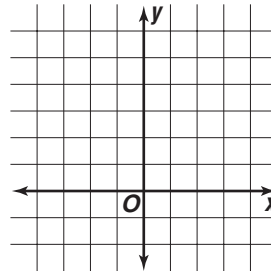
Graphing Systems of Inequalities

Solve each system of inequalities by graphing. If the system does not have a solution, write no solution.

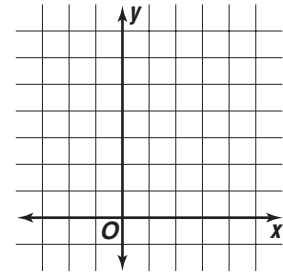
1. $x \leq 2$
 $y \geq -1$



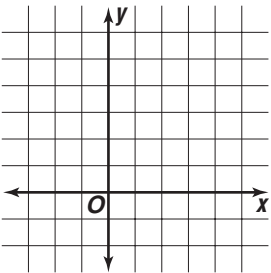
2. $x > 2$
 $y > x + 1$



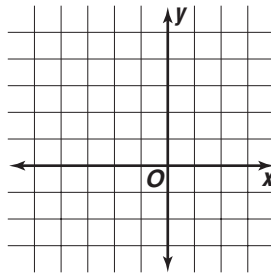
3. $x \geq 3$
 $y > x + 2$



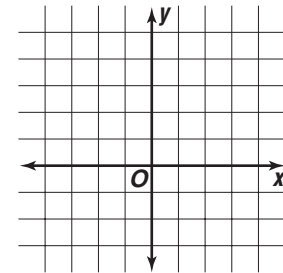
4. $x + y < 1$
 $y > x + 3$



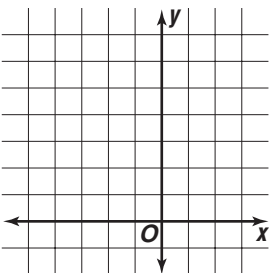
5. $2y \geq x + 4$
 $x - 2y \geq 1$



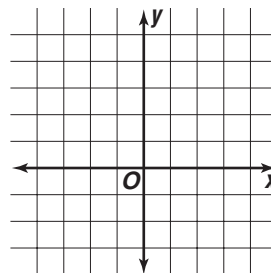
6. $y \leq x + 4$
 $x - y \leq 3$



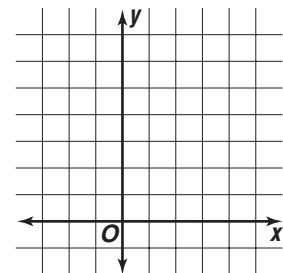
7. $x + y < 2$
 $y > x + 4$



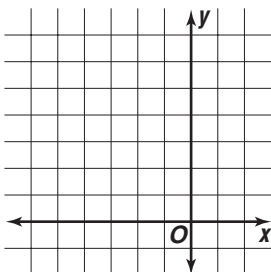
8. $x - y < -4$
 $y \leq x - 3$



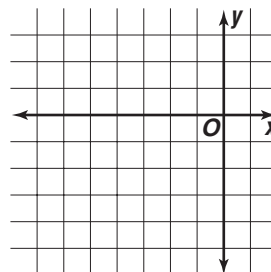
9. $y \geq x + 2$
 $y \leq 2x + 2$



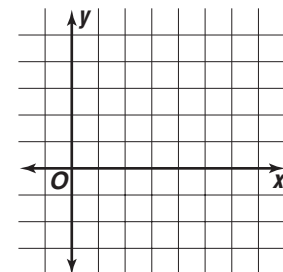
10. $x - y < -5$
 $y < -x + 1$



11. $y < x + 2$
 $x + y \geq -4$



12. $x + 2y > 5$
 $x - y > 1$

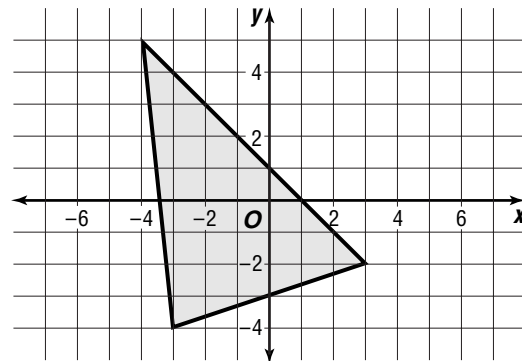


Enrichment

Describing Regions

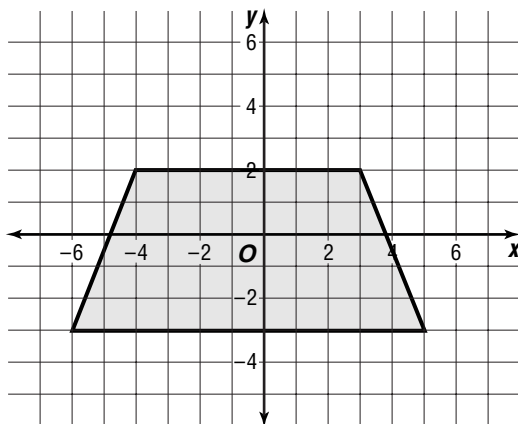
The shaded region inside the triangle can be described with a system of three inequalities.

$$\begin{aligned} y &< -x + 1 \\ y &> \frac{1}{3}x - 3 \\ y &> -9x - 31 \end{aligned}$$

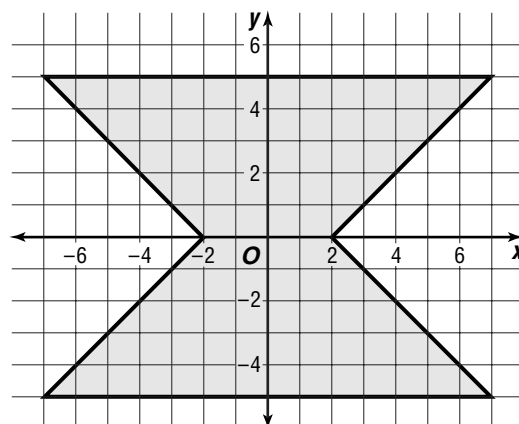


Write systems of inequalities to describe each region. You may first need to divide a region into triangles or quadrilaterals.

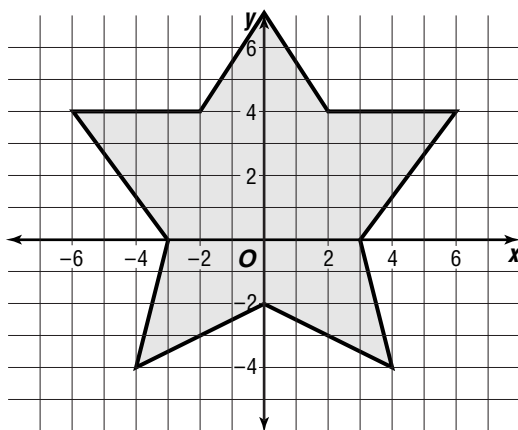
1.



2.



3.



Flower Offspring (Greenhouse Operator)

A hybrid results from merging two different kinds of plants. One satisfying aspect of horticulture is the achievement of a stunning new color.

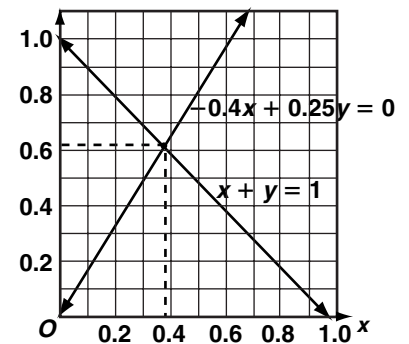
A greenhouse operator finds there is a 60% chance of getting pale red flowers and a 40% chance of getting bright red flowers from a seed from a pale red plant. There is a 25% chance of getting pale red flowers and a 75% chance of getting bright red flowers from a seed from a bright red plant. See the table at the right.

Start with	Probability of getting	
	Pale Red	Bright Red
Pale Red	0.6	0.4
Bright Red	0.25	0.75

Let x represent the overall probability of getting pale red flowers and y represent the overall probability of getting bright red flowers. A greenhouse operator can find these probabilities by solving the following system of equations.

$$\begin{aligned} (0.6 - 1)x + 0.25y &= 0 \\ x + y &= 1 \end{aligned}$$

Graph the system of equations.



The dashed lines indicate that the point of intersection is about (0.39, 0.61).

The probability of getting pale flowers is about 39% and the probability of getting bright red flowers is about 61%.

Solve.

- Suppose a greenhouse operator finds that there is a 40% chance of getting pale red flowers and a 60% chance of getting bright red flowers from a seed from a pale red plant. Also suppose that there is a 30% chance of getting pale red flowers and a 70% chance of getting bright red flowers from a seed from a bright red plant. Represent this data in the table at the right.
- The following system of equations can be used to find the probabilities of getting pale red flowers and bright red flowers in the long run.

Start with	Probability of getting	
	Pale Red	Bright Red
Pale Red		
Bright Red		

$$\begin{aligned} (0.4 - 1)x + 0.3y &= 0 \\ x + y &= 1 \end{aligned}$$

Graph the system of equations on the grid at the right to find the probabilities.

