NAME ______________________________ DATE __________ PERIOD ______

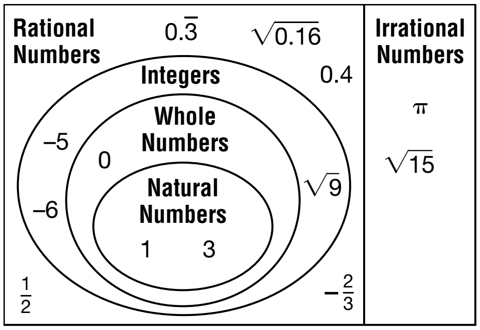
Study Guide

The Real Numbers

The numbers that we use every day belong to the set of real numbers. The real numbers can be divided into rational numbers and irrational numbers. Rational numbers can be expressed as fractions. Irrational numbers cannot be expressed as fractions. The set of rational numbers contains the natural numbers, the whole numbers, and the integers.

Natural Numbers: $\{1, 2, 3, 4, \ldots\}$
Whole Numbers: $\{0, 1, 2, 3, \ldots\}$
Integers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

The Venn diagram shows how subsets of the real numbers are related.



- 3 is a natural number, a whole number, an integer, and a rational number.
- 0 is a whole number, an integer, and a rational number.
- $\sqrt{9}$ or 3 is an integer and a rational number.
- $\sqrt{15}$ or 3.872983346… is an irrational number.
- $0.\overline{3}$ or $\frac{1}{3}$ is a rational number.

Example: Find an approximation to the nearest tenth of $\sqrt{12}$.

On a graphing calculator, press 2nd [$\sqrt{\ }$] 12 ENTER.

The result is 3.464101615. So an approximate value for $\sqrt{12}$ is 3.5.

Name the set or sets of numbers to which each real number belongs. Let N = natural numbers, W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. -4 **2.** $\frac{2}{5}$ **3.** $-\sqrt{25}$ **4.** 10

5. 2.3 6 $\sqrt{3}$ **7.** $-4.324781\ldots$ **8.** $-\frac{24}{8}$

9. $\sqrt{100}$ **10.** $\frac{1}{9}$ **11.** -0.25 **12.** $\sqrt{15}$

Find an approximation to the nearest tenth for each square root.

13. $\sqrt{2}$ **14.** $\sqrt{14}$ **15.** $-\sqrt{20}$ **16.** $\sqrt{55}$

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Practice

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The Real Numbers

Name the set or sets of numbers to which each real number belongs. Let N = natural numbers, W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. $\sqrt{19}$ **2.** -8 **3.** 1.737337... **4.** $0.\overline{4}$

5. $-\frac{5}{6}$ **6.** $\sqrt{64}$ **7.** $-\frac{28}{7}$ **8.** $-\sqrt{144}$

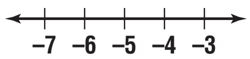
9. 0.414114111... **10.** $\frac{1}{3}$ **11.** 13 **12.** 0.75

Find an approximation, to the nearest tenth, for each square root. Then graph the square root on a number line.

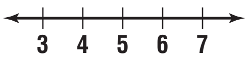
13. $\sqrt{6}$

14. $\sqrt{11}$

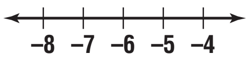
15. $-\sqrt{24}$



16. $\sqrt{30}$

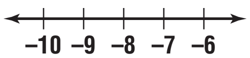


17. $-\sqrt{38}$

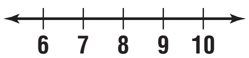


18. $\sqrt{51}$

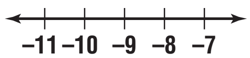
19. $-\sqrt{65}$



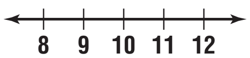
20. $\sqrt{72}$



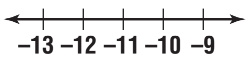
21. $-\sqrt{89}$



22. $\sqrt{118}$



23. $-\sqrt{131}$



24. $\sqrt{104}$

Determine whether each number is **rational** ***or*** **irrational**. ***If it is irrational, find two consecutive integers between which its graph lies on the number line.***

25. $\sqrt{28}$ **26.** $-\sqrt{9}$ **27.** $\sqrt{56}$

28. $-\sqrt{14}$ **29.** $\sqrt{36}$ **30.** $\sqrt{99}$

31. $-\sqrt{73}$ **32.** $\sqrt{196}$ **33.** $\sqrt{77}$

34. $-\sqrt{100}$ **35.** $\sqrt{88}$ **36.** $-\sqrt{46}$

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14–1 Enrichment

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Roots

The symbol $\sqrt{}$ indicates a square root. By placing a number in the upper left, the symbol can be changed to indicate higher roots.

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[4]{81} = 3$ because $3^4 = 81$

$\sqrt[5]{100{,}000} = 10$ because $10^5 = 100{,}000$

Find each of the following.

1. $\sqrt[3]{125}$

2. $\sqrt[4]{16}$

3. $\sqrt[8]{1}$

4. $\sqrt[3]{27}$

5. $\sqrt[5]{32}$

6. $\sqrt[3]{64}$

7. $\sqrt[3]{1000}$

8. $\sqrt[3]{216}$

9. $\sqrt[6]{1{,}000{,}000}$

10. $\sqrt[3]{1{,}000{,}000}$

11. $\sqrt[4]{256}$

12. $\sqrt[3]{729}$

13. $\sqrt[6]{64}$

14. $\sqrt[4]{625}$

15. $\sqrt[5]{243}$

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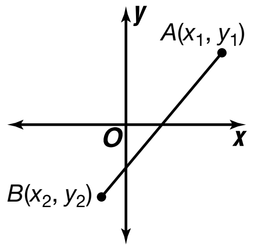
Study Guide

The Distance Formula

You can use the Distance Formula to find the distance between two points on the coordinate plane.

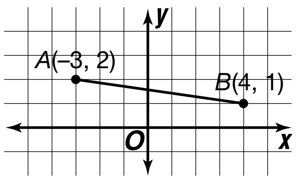
The Distance Formula

The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example: Find the distance between $A(-3, 2)$ and $B(4, 1)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-3))^2 + (1 - 2)^2} \\ &= \sqrt{7^2 + (-1)^2} \\ &= \sqrt{49 + 1} \\ &= \sqrt{50} \text{ or about 7.1 units} \end{aligned}$$



Find the distance between each pair of points. Round to the nearest tenth, if necessary.

1. $P(-4, 0)$, $Q(5, 0)$

2. $X(0, 0)$, $Y(6, 8)$

3. $J(-5, 1)$, $K(-2, 5)$

4 $M(-4, -14)$, $N(3, 10)$

5. $R(-7, 4)$, $S(2, -1)$

6 $C(0, -5)$, $D(3, 2)$

7. $X(5, 9)$, $Y(2, 3)$

8. $A(-9, 1)$, $B(-8, 3)$

9. $G(5, -4)$, $H(10, 6)$

10. $U(7, -3)$, $V(-3, -2)$

11. $M(1, 3)$, $N(-1, 4)$

12. $X(12, -3)$, $Y(7, -15)$

13. $A(1, 20)$, $B(12, -4)$

14. $E(-3, -3)$, $F(-8, -8)$

15. $A(5, 4)$, $B(0, 6)$

16. $S(-6, -6)$, $T(-5, -1)$

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Practice

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The Distance Formula

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

1. $X(4, 2)$, $Y(8, 6)$

2. $Q(-3, 8)$, $R(2, -4)$

3. $A(0, -3)$, $B(-6, 5)$

4. $M(-9, -5)$, $N(-4, 1)$

5. $J(6, 2)$, $K(-7, 5)$

6. $S(-2, 4)$, $T(-3, 8)$

7. $V(-1, -2)$, $W(-9, -7)$

8. $O(5, 2)$, $P(7, -4)$

9. $G(3, 4)$, $H(-2, 1)$

Find the value of a if the points are the indicated distance apart.

10. $C(1, 1)$, $D(a, 7)$; $d = 10$

11. $Y(a, 3)$, $Z(5, -1)$; $d = 5$

12. $F(3, -2)$, $G(-9, a)$; $d = 13$

13. $W(-2, a)$, $X(7, -4)$; $d = \sqrt{85}$

14. $B(a, -6)$, $C(8, -3)$; $d = \sqrt{34}$

15. $T(2, 2)$, $U(a, -4)$; $d = \sqrt{72}$

14–2

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Enrichment

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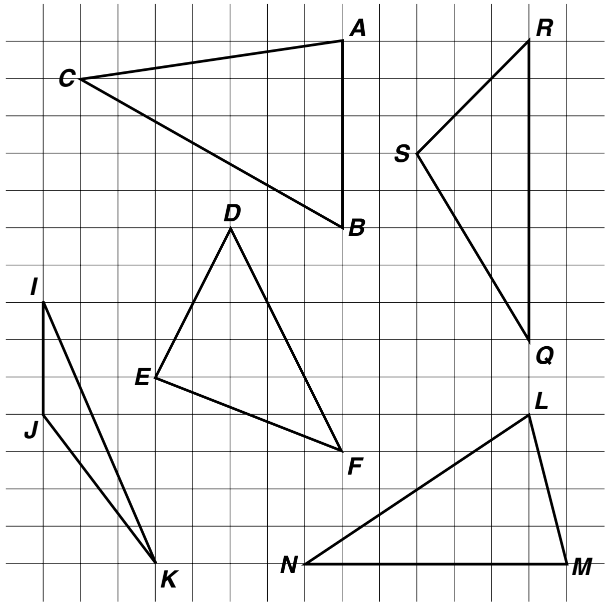
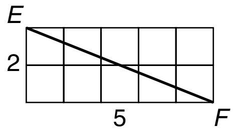
Lengths on a Grid

You can easily find segment lengths on a grid if the endpoints are grid-line intersections. For horizontal or vertical segments, simply count squares. For diagonal segments, use the Pythagorean Theorem. This theorem states that in any right triangle, if the length of the longest side (the side opposite the right angle) is c and the two shorter sides have lengths a and b, then $c^2 = a^2 + b^2$.



Example: Find the measure of $\overline{EF}$ on the grid at the right. Locate a right triangle with $\overline{EF}$ as its longest side.



$$EF = \sqrt{2^2 + 5^2}$$
$$= \sqrt{29}$$
$$\approx 5.4 \text{ units}$$

Find each measure to the nearest tenth of a unit.

1. IJ **2.** MN **3.** RS **4.** QS

5. IK **6.** JK **7.** LM **8.** LN

Use the grid above. Find the perimeter of each triangle to the nearest tenth of a unit.

9. $\triangle ABC$ **10.** $\triangle QRS$ **11.** $\triangle DEF$ **12.** $\triangle LMN$

13. Of all the segments shown on the grid, which is longest? What is its length?

14. On the grid, 1 unit = 0.5 cm. How can the answers above be used to find the measures in centimeters?

15. Use your answer from Exercise 8 to calculate the length of segment LN in centimeters. Check by measuring with a centimeter ruler.

16. Use a centimeter ruler to find the perimeter of triangle IJK to the nearest tenth of a centimeter.

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Study Guide

Simplifying Radical Expressions

To simplify a radical expression such as $\sqrt{54}$, first identify any perfect square factors of the radicand. Then apply the Product Property of Square Roots.

$$\begin{aligned}\sqrt{54} &= \sqrt{6 \cdot 9} && \textit{3}^2 = 9\textit{, so 9 is a perfect square factor of 54.}\\ &= \sqrt{6} \cdot \sqrt{9} && \textit{Product Property of Square Roots}\\ &= \sqrt{6} \cdot 3 \text{ or } 3\sqrt{6} && \textit{Simplify } \sqrt{9}.\end{aligned}$$

To simplify radical expressions such as $\frac{\sqrt{24}}{\sqrt{2}}$ and $\frac{\sqrt{6}}{\sqrt{5}}$ that involve division, you must eliminate the radicals in the denominator. To do so, you can use the Quotient Property of Square Roots and a method called **rationalizing the denominator**. Rationalizing the denominator involves multiplying the fraction by a special form of 1.

Examples: Simplify each expression.

a. $\frac{\sqrt{24}}{\sqrt{2}}$

$$\begin{aligned}\frac{\sqrt{24}}{\sqrt{2}} &= \sqrt{\frac{24}{2}} && \textit{Quotient Property}\\ &= \sqrt{12} && 24 \div 2 = 12\\ &= \sqrt{3 \cdot 2^2} && 3 \cdot 4 = 3 \cdot 2^2\\ &= 2\sqrt{3} && \sqrt{2^2} = 2\end{aligned}$$

b. $\frac{\sqrt{6}}{\sqrt{5}}$

$$\begin{aligned}\frac{\sqrt{6}}{\sqrt{5}} &= \frac{\sqrt{6}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} && \frac{\sqrt{5}}{\sqrt{5}} = 1\\ &= \frac{\sqrt{6 \cdot 5}}{\sqrt{5 \cdot 5}} && \textit{Product Property}\\ &= \frac{\sqrt{30}}{\sqrt{25}} && \textit{Quotient Property}\\ &= \frac{\sqrt{30}}{5} && \textit{Simplify.}\end{aligned}$$

Simplify each expression. Leave in radical form.

1. $\sqrt{28}$ **2.** $\sqrt{48}$ **3.** $\sqrt{50}$ **4.** $\sqrt{8}$

5. $\sqrt{99}$ **6.** $\sqrt{12} \cdot \sqrt{3}$ **7.** $\sqrt{8} \cdot \sqrt{6}$ **8.** $2\sqrt{3} \cdot \sqrt{3}$

9. $\frac{\sqrt{18}}{\sqrt{6}}$ **10.** $\frac{\sqrt{75}}{\sqrt{3}}$ **11.** $\frac{\sqrt{49}}{\sqrt{7}}$ **12.** $\frac{\sqrt{72}}{\sqrt{36}}$

13. $\frac{\sqrt{3}}{\sqrt{4}}$ **14.** $\frac{\sqrt{5}}{\sqrt{3}}$ **15.** $\frac{\sqrt{12}}{\sqrt{5}}$ **16.** $\frac{\sqrt{6}}{\sqrt{8}}$

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Practice

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Simplifying Radical Expressions

Simplify each expression. Leave in radical form.

1. $\sqrt{28}$ **2.** $\sqrt{48}$ **3.** $\sqrt{72}$

4. $\sqrt{90}$ **5.** $\sqrt{175}$ **6.** $\sqrt{245}$

7. $\sqrt{7} \cdot \sqrt{14}$ **8.** $\sqrt{2} \cdot \sqrt{10}$ **9.** $\sqrt{10} \cdot \sqrt{60}$

10. $\frac{\sqrt{48}}{\sqrt{2}}$ **11.** $\frac{\sqrt{54}}{\sqrt{3}}$ **12.** $\frac{\sqrt{96}}{\sqrt{8}}$

13. $\frac{\sqrt{20}}{\sqrt{3}}$ **14.** $\frac{\sqrt{2}}{\sqrt{10}}$ **15.** $\frac{\sqrt{8}}{\sqrt{6}}$

16. $\frac{5}{4 - \sqrt{7}}$ **17.** $\frac{4}{3 + \sqrt{2}}$ **18.** $\frac{3}{3 - \sqrt{3}}$

Simplify each expression. Use absolute value symbols if necessary.

19. $\sqrt{50x^2}$ **20.** $\sqrt{27ab^3}$ **21.** $\sqrt{49c^6d^4}$

22. $\sqrt{63x^2y^5z^2}$ **23.** $\sqrt{56m^2n^4p^3}$ **24.** $\sqrt{108r^2s^3t^6}$

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14–3 Enrichment

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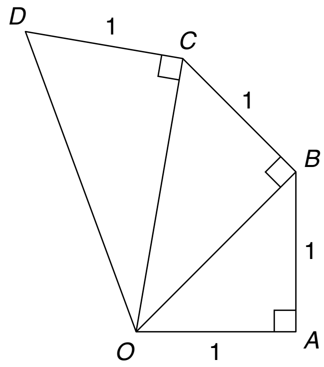
Pages 614–619

The Wheel of Theodorus



The Greek mathematicians were intrigued by problems of representing different numbers and expressions using geometric constructions.

Theodorus, a Greek philosopher who lived about 425 B.C., is said to have discovered a way to construct the sequence $\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \cdots$.

The beginning of his construction is shown. You start with an isosceles right triangle with sides 1 unit long.

Use the figure above. Write each length as a radical expression in simplest form.

1. line segment *AO*

2. line segment *BO*

3. line segment *CO*

4. line segment *DO*

5. Describe how each new triangle is added to the figure.

6. The length of the hypotenuse of the first triangle is $\sqrt{2}$. For the second triangle, the length is $\sqrt{3}$. Write an expression for the length of the hypotenuse of the *n*th triangle.

7. Show that the method of construction will always produce the next number in the sequence. (*Hint:* Find an expression for the hypotenuse of the $(n + 1)$th triangle.)

8. In the space below, construct a Wheel of Theodorus. Start with a line segment 1 centimeter long. When does the Wheel start to overlap?

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Study Guide

Adding and Subtracting Radical Expressions

You can use the Distributive Property to add and subtract radical expressions with the same radicand.

Example 1: Simplify each expression.

a. $2\sqrt{3} + 8\sqrt{3}$

$2\sqrt{3} + 8\sqrt{3} = (2 + 8)\sqrt{3}$ *Distributive Property*

$= 10\sqrt{3}$ *Simplify.*

b. $15\sqrt{10} - 2\sqrt{10}$

$15\sqrt{10} - 2\sqrt{10} = (15 - 2)\sqrt{10}$ *Distributive Property*

$= 13\sqrt{10}$ *Simplify.*

Recall that when you add monomials, only like terms can be combined. The same is true when you add or subtract radical expressions. Radical expressions are like terms if they have the same radicand when they are in simplest form.

Example 2: Simplify $4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7}$.

$4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7} = 4\sqrt{2} + \left(6\sqrt{7} - 11\sqrt{7}\right)$ *Group like terms.*

$= 4\sqrt{2} + (6 - 11)\sqrt{7}$ *Distributive Property*

$= 4\sqrt{2} - 5\sqrt{7}$ *Simplify.*

Simplify each expression.

1. $8\sqrt{5} + 8\sqrt{5}$

2. $5\sqrt{11} - 3\sqrt{11}$

3. $-9\sqrt{2} + \sqrt{2}$

4. $-3\sqrt{3} - 10\sqrt{3}$

5. $4\sqrt{6} + \sqrt{6}$

6. $\sqrt{10} - 6\sqrt{10} + 7\sqrt{10}$

7. $2\sqrt{2} - 5\sqrt{2} + 4\sqrt{5}$

8. $\sqrt{11} - 15\sqrt{3} - 10\sqrt{3}$

9. $8\sqrt{13} + 3\sqrt{13} - 4\sqrt{7} - 3\sqrt{7}$

10. $-4\sqrt{6} + 2\sqrt{6} + \sqrt{3} + \sqrt{3}$

11. $-3\sqrt{5} + 9\sqrt{2} + 5\sqrt{2} + 5\sqrt{5}$

12. $7\sqrt{2} + \sqrt{8}$

13. $3\sqrt{3} + \sqrt{27}$

14. $5\sqrt{32} - 6\sqrt{2}$

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Practice

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Pages 620–623

Adding and Subtracting Radical Expressions

Simplify each expression.

1. $3\sqrt{7} + 4\sqrt{7}$

2. $9\sqrt{2} - 4\sqrt{2}$

3. $-5\sqrt{17} + 12\sqrt{17}$

4. $7\sqrt{3} - 3\sqrt{3}$

5. $-8\sqrt{5} + 2\sqrt{5}$

6. $-7\sqrt{11} - 2\sqrt{11}$

7. $13\sqrt{10} - 5\sqrt{10}$

8. $-6\sqrt{7} + 4\sqrt{7}$

9. $3\sqrt{7} + \sqrt{3}$

10. $2\sqrt{6} + 4\sqrt{6} + 5\sqrt{6}$

11. $5\sqrt{3} + 4\sqrt{3} - 7\sqrt{3}$

12. $3\sqrt{2} - 2\sqrt{2} + 5\sqrt{2}$

13. $11\sqrt{5} - 3\sqrt{5} - 2\sqrt{5}$

14. $6\sqrt{13} + 3\sqrt{13} - 12\sqrt{13}$

15. $4\sqrt{10} - 3\sqrt{10} - 5\sqrt{10}$

16. $4\sqrt{6} - 2\sqrt{6} + 3\sqrt{6}$

17. $7\sqrt{7} + 4\sqrt{3} - 5\sqrt{7}$

18. $-9\sqrt{2} + 4\sqrt{6} + 2\sqrt{2}$

19. $\sqrt{12} + 2\sqrt{27}$

20. $5\sqrt{63} - \sqrt{28}$

21. $-4\sqrt{96} + 6\sqrt{24}$

22. $-3\sqrt{45} + 3\sqrt{180}$

23. $-4\sqrt{56} + 3\sqrt{126}$

24. $2\sqrt{72} - 3\sqrt{50}$

25. $7\sqrt{32} + 3\sqrt{75}$

26. $\sqrt{32} + \sqrt{8} + \sqrt{18}$

27. $2\sqrt{20} - \sqrt{80} + \sqrt{45}$

Enrichment

Student Edition
Pages 620–623

Other Kinds of Means

There are many different kinds of means besides the arithmetic mean. A mean for a set of numbers has these two properties:

a. It typifies or represents the set.

b. It is not less than the least number and it is not greater than the greatest number.

Here are the formulas for the arithmetic mean and three other means.

Arithmetic Mean
Add the numbers in the set. Then divide the sum by n, the number of elements in the set.

$$\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Geometric Mean
Multiply all the numbers in the set. Then find the nth root of their product.

$$\sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot \cdots \cdot x_n}$$

Harmonic Mean
Divide the number of elements in the set by the sum of the reciprocals of the numbers.

$$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}$$

Quadratic Mean
Add the squares of the numbers. Divide their sum by the number in the set. Then, take the square root.

$$\sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2}{n}}$$

Find the four different means for each set of numbers.

1. 10, 100

2. 50, 60

3. 1, 2, 3, 4, 5,

4. 2, 2, 4, 4

5. Use the results from Exercises 1 to 4 to compare the relative sizes of the four types of means.

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Study Guide

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Solving Radical Equations

Equations that contain radicals are called **radical equations**.

Steps for Solving Radical Equations
1. Isolate the radical on one side of the equation. **2.** Square each side of the equation to eliminate the radical. **3.** Check all solutions. Reject any solutions that do not satisfy the original equation.

Examples: Solve each equation. Check your solution.

a. $\sqrt{x} - 2 = 7$

$$\sqrt{x} - 2 = 7$$
$$\sqrt{x} - 2 + 2 = 7 + 2 \quad \textit{Add 2 to each side.}$$
$$\sqrt{x} = 9$$
$$(\sqrt{x})^2 = 9^2 \quad \textit{Square each side.}$$
$$x = 81$$

Check: $\sqrt{x} - 2 = 7$

$$\sqrt{81} - 2 \stackrel{?}{=} 7 \quad \textit{Replace x with 81.}$$
$$9 - 2 \stackrel{?}{=} 7$$
$$7 = 7 \checkmark$$

The solution is 81.

b. $\sqrt{x+1} + 5 = 4$

$$\sqrt{x+1} + 5 = 4$$
$$\sqrt{x+1} = -1 \quad \textit{Subtract 5.}$$
$$(\sqrt{x+1})^2 = (-1)^2 \quad \textit{Square each side.}$$
$$x + 1 = 1$$
$$x = 0$$

Check: $\sqrt{x+1} + 5 = 4$

$$\sqrt{0+1} + 5 \stackrel{?}{=} 4$$
$$1 + 5 \stackrel{?}{=} 4$$
$$6 \neq 4$$

There is no solution.

Some radical equations have no solution when the domain is the set of real numbers. Example b has no solution because the square root of $x + 1$ cannot be negative.

Solve each equation. Check your solution.

1. $\sqrt{x} = 7$

2. $\sqrt{x} = 2$

3. $\sqrt{x} = -9$

4. $\sqrt{x} - 9 = -1$

5. $\sqrt{x} + 12 = 6$

6. $\sqrt{x} + 2 = 4$

7. $\sqrt{x} - 8 = -3$

8. $\sqrt{x} - 1 = -5$

9. $\sqrt{x+4} = 3$

10. $2 = \sqrt{x-5}$

11. $\sqrt{x-2} + 1 = 6$

12. $\sqrt{x+3} - 7 = 0$

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14–5 Practice

Student Edition
Pages 624–629

Solving Radical Equations

Solve each equation. Check your solution.

1. $\sqrt{x} - 6 = 3$

2. $\sqrt{k} + 7 = 20$

3. $\sqrt{p + 3} = 3$

4. $\sqrt{n + 11} = 5$

5. $\sqrt{w - 2} - 1 = 6$

6. $\sqrt{y - 5} + 9 = 14$

7. $\sqrt{2r + 1} - 10 = -1$

8. $\sqrt{3h - 11} + 2 = 9$

9. $\sqrt{a + 4} = a - 8$

10. $\sqrt{z - 3} + 5 = z$

11. $\sqrt{3b + 9} + 3 = b$

12. $\sqrt{5f - 5} + 1 = f$

13. $\sqrt{8 + 2c} = c - 8$

14. $\sqrt{3s - 6} = s - 2$

15. $\sqrt{4h + 4} + h = 7$

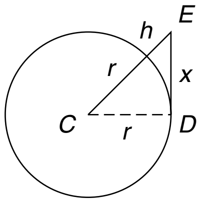
16. $\sqrt{5m + 4} = m + 2$

17. $\sqrt{2y - 7} - y = -5$

18. $\sqrt{3k + 4} + k = 8$

Enrichment

Using Radical Equations



The circle with center C and radius r represents Earth. If your eye is at point E, the distance to the horizon is x, or the length of tangent segment ED. The segment drawn from C to D forms a right angle with ED. Thus, $\triangle EDC$ is a right triangle. Apply the Pythagorean Theorem.

$$(\text{length of } CD)^2 + (\text{length of } ED)^2 = (\text{length of } EC)^2$$
$$r^2 + x^2 = (r + h)^2$$
$$x^2 = (r + h)^2 - r^2$$
$$x = \sqrt{(r + h)^2 - r^2}$$

1. Show that this equation is equivalent to $x = \sqrt{2r + h} \cdot \sqrt{h}$.

If the distance h is very small compared to r, $\sqrt{2r + h}$ is close to $\sqrt{2r}$.

$$x \approx \sqrt{2r} \cdot \sqrt{h}$$

The radius of Earth is about 20,900,000 feet.

$\sqrt{2r} \approx \sqrt{2(20{,}900{,}000)} = \sqrt{41{,}800{,}000} \approx 6465$ feet. Thus, $x \approx 6465\sqrt{h}$.

If you are h feet above Earth, you are about $6465\sqrt{h}$ feet from the horizon. Since there are 5280 feet in one mile,

$$6465\sqrt{h} \text{ feet} = \frac{5465\sqrt{h}}{5280} \text{ miles} \approx 1.22\sqrt{h} \text{ miles.}$$

Thus, if you are h feet above Earth's surface, you can see $1.22\sqrt{h}$ miles in any direction.

2. How far can you see to the nearest mile if your eye is:

a. 1454 feet above the ground (the height of the Sears Tower)?

b. 30,000 feet above the ground (altitude for a commercial airliner)?

c. $5\frac{1}{2}$ feet from the ground?

A strong wind can severely alter the effect of an actual temperature on the human body. For example, a temperature of 32°F is much more dangerous on a windy day than on a still day. Windchill is a temperature value assigned to a particular combination of wind speed and temperature. If w is the speed of the wind in miles per hour and t is the actual temperature in degrees Fahrenheit, then the approximate windchill temperature in °F is given by the formula

$$\text{windchill temperature} = 92.4 - \frac{(6.91\sqrt{w} + 10.45 - 0.477w)(91.4 - t)}{22.1}.$$

This formula gives reasonably accurate results for the windchill temperature when $5 \le w \le 30$ and $-30 \le t \le 50$.

3. Find the windchill temperature to the nearest degree when the actual temperature is -15°F and the wind speed is
a. 10 mi/h **b.** 20 mi/h **c.** 30 mi/h

4. Find the windchill temperature to the nearest degree when the wind speed is 20 mi/h and the actual temperature is
a. 30°F **b.** 0°F **c.** -30°F

NAME ______________________ DATE __________ PERIOD ______

School-to-Workplace

Student Edition
Pages 606–611

City Water Supplies (Water Supply Technician)

Water is a precious resource that, all too often, we take for granted. Without careful control of the allocation of a water supply, we may find that we do not have enough water when we need it.

In conjunction with city planners, waste treatment managers, and fire department officials, water supply technicians help assure the adequacy of clean water.

There is a relationship between a city's capacity to supply water to its citizens and the city's size. Suppose that a city has a population P (in thousands). Then the number of gallons per minute that are required to assure water adequacy is given by the expression below.

$$1020\sqrt{P}\,(1 - 0.01\sqrt{P})$$

If a city has a population of 55,000 people, how many gallons per minute must the city's pumping stations be able to supply?

Find $1020\sqrt{55}\,(1 - 0.01\sqrt{55})$.

Use a calculator.

1020 [×] [2nd] [√‾] 55 [)] [×] [(] 1 [−] .01 [×] [2nd] [√‾] 55 [)] [=] 7003.522457

The city's water system must be capable of a water flow greater than 7000 gallons per minute.

Solve.

1. A city has about 37,550 people, how many gallons per minute must the city's pumps be able to supply?

2. A city has about 63,350 people. If firefighters are using 1250 gallons per minute at a fire scene, how many gallons per minute are available for other purposes?

3. A city had a population of 43,250 people five years ago. Since then the population increased by 2250 people. How many more gallons per minute had to be added to the city's pumping capacity?