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Study Guide

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Pages 638–643

Simplifying Rational Expressions

A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials. For example, $\frac{3}{x}$, $\frac{3}{x-2}$, $\frac{x-2}{3}$, and $\frac{x}{x-2}$ are all rational expressions because 3, x, and $x - 2$ are all polynomials.

To simplify a rational expression, use the same steps that you use to simplify any fraction.

- First factor the numerator and denominator. The factors may be polynomials.
- Then divide the numerator and denominator by the greatest common factor. The greatest common factor may be a polynomial.

Example 1: Simplify $\frac{16a^4b^2}{20a^2b^5}$.

$$\frac{16a^4b^2}{20a^2b^5} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b}{2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b}$$

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot a \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 5 \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot b \cdot b \cdot b}$$ *The GCF is $4a^2b^2$.*

$$= \frac{4a^2}{5b^3}$$

Example 2: Simplify $\frac{m(m+1)}{m^2 - 5m - 6}$.

$$\frac{m(m+1)}{m^2 - 5m - 6} = \frac{m(m+1)}{(m-6)(m+1)}$$ *Factor $m^2 - 5m - 6$.*

$$= \frac{m\overset{1}{\cancel{(m+1)}}}{(m-6)\underset{1}{\cancel{(m+1)}}}$$ *The GCF is $(m + 1)$.*

$$= \frac{m}{m-6}$$

Simplify each rational expression.

1. $\frac{10}{25}$

2. $\frac{3m}{6n}$

3. $\frac{12xy}{6x}$

4. $\frac{-8abc}{16ac}$

5. $\frac{6a^2b^3}{-2a^4b^3}$

6. $\frac{3(x+2)}{6(x+2)}$

7. $\frac{m-4}{m(m-4)}$

8. $\frac{d(d-4)}{c(d-4)}$

9. $\frac{(c-2)(d+3)}{(c-2)(d+6)}$

10. $\frac{x^2 - 2x}{x-2}$

11. $\frac{y^2 + 3y}{y(y-3)}$

12. $\frac{x+3}{x^2 + x - 6}$

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Simplifying Rational Expressions

Find the excluded value(s) for each rational expression.

1. $\frac{2n}{n - 4}$

2. $\frac{6}{x + 3}$

3. $\frac{3b}{b(b + 9)}$

4. $\frac{y + 2}{y^2 - 4}$

5. $\frac{4x + 6}{(x + 6)(x - 5)}$

6. $\frac{2a - 2}{a^2 - 3a - 28}$

Simplify each rational expression.

7. $\frac{6}{15}$

8. $\frac{12m}{18m^3}$

9. $\frac{16x^2y}{36xy^3}$

10. $\frac{25ab}{30b^2}$

11. $\frac{-8y^4z}{20y^6z^2}$

12. $\frac{5(x - 1)}{8(x - 1)}$

13. $\frac{y(y + 7)}{9(y + 7)}$

14. $\frac{x^2 - 4x}{3(x - 4)}$

15. $\frac{x^2 + 2x}{5x + 10}$

16. $\frac{x^2 + 5x}{(x + 5)(x - 7)}$

17. $\frac{x^2 - 6x}{x^2 - 4x - 12}$

18. $\frac{(x + 4)(x + 4)}{(x + 4)(x - 2)}$

19. $\frac{b^2 + 6b + 9}{b^2 - 2b - 15}$

20. $\frac{y^2 - 36}{y^2 + 9y + 18}$

21. $\frac{x^2 - 16}{x^2 + x - 12}$

22. $\frac{y^2 + 4y + 4}{y^2 - 4}$

23. $\frac{a^2 + 3a}{a^2 - 3a - 18}$

24. $\frac{y^2 + 7y + 10}{y^2 + 5y}$

25. $\frac{x^2 + 4x + 3}{x^2 + 3x + 2}$

26. $\frac{x^2 - 6x + 8}{x^2 + x - 6}$

27. $\frac{9 - x^2}{x^2 + 6x - 27}$

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Enrichment

Division by Zero?

You may remember being told, "division by zero is not possible" or "division by zero is undefined" or "we never divide by zero." Have you wondered why this is so? Consider the two equations below.

$$\frac{5}{0} = n \qquad \frac{0}{0} = m$$

Because multiplication is the inverse of division, these lead to the following.

$$0 \cdot n = 5 \qquad 0 \cdot m = 0$$

There is no number that will make the first equation true. Any number at all will satisfy the second equation.

For each expression, give the values that must be excluded from the replacement set in order to prevent division by zero.

1. $\frac{x + 1}{x - 1}$

2. $\frac{2(x + 1)}{2x - 1}$

3. $\frac{(x + 1)(x - 1)}{(x + 2)(x - 2)}$

4. $\frac{x + y + 3}{(3x - 1)(3y - 1)}$

5. $\frac{x^2 + y^2 + z^2}{2xyz}$

6. $\frac{(x + y)^2}{x - y}$

Many demonstrations or "proofs" that lead to impossible results include a step involving division by zero. Explain what is wrong with each "proof" below.

7. $0 \cdot 1 = 0$ and $0 \cdot 2 = 0$.
Therefore, $\frac{0}{0} = 1$ and $\frac{0}{0} = 2$.
Therefore, $1 = 2$.

8. Assume that $a = b$.
Then $ab = a^2$
Therefore, $ab - b^2 = a^2 - b^2$.
Next it is shown that $a^2 - b^2 = (a + b)(a - b)$.
$(a + b)(a - b) = (a + b)\,a - (a + b)b$
$= a^2 + ba - ab - b^2$
$= a^2 + 0 - b^2$
$= a^2 - b^2$
Therefore, $ab - b^2 = (a + b)(a - b)$.
Also, $b(a - b) = ba - b^2 = ab - b^2$.
Therefore, $b(a - b) = (a + b)(a - b)$.
Therefore, $b = a + b$.
Therefore, $b = 2b$.
Therefore, $1 = 2$.

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Multiplying and Dividing Rational Expressions

To multiply rational expressions, simplify the expressions first, then multiply.

Example 1: Find $\frac{12a^2}{18b^2} \cdot \frac{9ab}{12a^4}$.

$$\frac{12a^2}{18b^2} \cdot \frac{9ab}{12a^4} = \frac{\overset{1}{\cancel{12}}\overset{1}{\cancel{a^2}}}{\underset{2}{\cancel{18}}\underset{b}{\cancel{b^2}}} \cdot \frac{\overset{1}{\cancel{9}}\overset{1}{\cancel{a}}\overset{1}{\cancel{b}}}{\underset{1}{\cancel{12}}\underset{a}{\cancel{a^4}}}$$

$$= \frac{1}{2ab}$$

To divide a rational expression by a nonzero rational expression, multiply by its reciprocal. Simplify the expressions, if necessary.

Example 2: Find $\frac{3m + 6}{m - 4} \div (m + 2)$.

$$\frac{3m + 6}{m - 4} \div (m + 2) = \frac{3m + 6}{m - 4} \cdot \frac{1}{m + 2}$$

The reciprocal of $(m + 2)$ is $\frac{1}{m + 2}$.

$$= \frac{3(m + 2)}{m - 4} \cdot \frac{1}{m + 2}$$

$$= \frac{3\overset{1}{\cancel{(m + 2)}}}{m - 4} \cdot \frac{1}{\underset{1}{\cancel{m + 2}}}$$

$$= \frac{3}{m - 4}$$

Find each product or quotient.

1. $\frac{4x}{3y} \cdot \frac{y^3}{8}$

2. $\frac{8a^2}{12b^2} \cdot \frac{18b}{4a}$

3. $\frac{2(x - y)}{x} \cdot \frac{x^2}{x - y}$

4. $\frac{6(m - n)}{7} \cdot \frac{7}{12(m - n)}$

5. $\frac{x^2}{y^2} \div \frac{x}{y}$

6. $\frac{m + n}{4} \div \frac{m + n}{6}$

7. $\frac{x - 4}{x} \div \frac{1}{x^2}$

8. $\frac{7x}{x + 3} \div \frac{21}{2x + 6}$

9. $\frac{2m + 4}{m - 3} \div (m + 2)$

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Practice

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Multiplying and Dividing Rational Expressions

Find each product.

1. $\frac{3x^2}{2y} \cdot \frac{y^2}{9}$

2. $\frac{4a^2b}{6b^2c} \cdot \frac{3ab}{2c}$

3. $\frac{7n}{n-2} \cdot \frac{3(n-2)}{28}$

4. $\frac{2}{m(m+3)} \cdot \frac{3m+9}{6}$

5. $\frac{4y+8}{y^2-2y} \cdot \frac{y-2}{y+2}$

6. $\frac{x^2-49}{x^2+5x} \cdot \frac{x+5}{x+7}$

7. $\frac{5x+25}{x^2-5x+6} \cdot \frac{x-3}{x+5}$

8. $\frac{a+5}{3a+6} \cdot \frac{3a^2+6a}{a^2+2a-15}$

9. $\frac{x^2+8x+12}{4x-12} \cdot \frac{2x-6}{x^2+4x-12}$

10. $\frac{2n^2-10n}{n^2-9n+20} \cdot \frac{n^2-8n+16}{4n^2}$

Find each quotient.

11. $\frac{4a^3}{b^2c} \div \frac{2a}{bc}$

12. $\frac{15x^2y^2}{3} \div 3xy$

13. $\frac{3y+9}{y+2} \div (y+3)$

14. $\frac{8n^3}{n-4} \div \frac{4n}{n-4}$

15. $\frac{6x^2y}{3y} \div 2xy$

16. $\frac{b^2-81}{b} \div (b+9)$

17. $\frac{6x^2+36x}{4x} \div \frac{4x+24}{2x^2}$

18. $\frac{y^2+5y-14}{9y} \div \frac{y^2-8y+12}{3y^2}$

19. $\frac{x^2-2x-15}{x-2} \div \frac{x^2-10x+25}{2-x}$

20. $\frac{y^2-8y+7}{5y^2} \div \frac{7-y}{10y}$

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Enrichment

Complex Fractions

Complex fractions are really not complicated. Remember that a fraction can be interpreted as dividing the numerator by the denominator.

$$\frac{\frac{2}{3}}{\frac{5}{7}} = \frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{2(7)}{3(5)} = \frac{14}{15}$$

Let a, b, c, and d be numbers, with $b \neq 0$, $c \neq 0$, and $d \neq 0$.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Notice the pattern: *numerator of the answer* (ad) $\frac{\frac{a}{b}}{\frac{c}{d}}$ *denominator of the answer* (bc)

Example 1: Simplify $\dfrac{\frac{5x}{4}}{\frac{x+2}{3}}$.

$$\frac{\frac{5x}{4}}{\frac{x+2}{3}} = \frac{5x(3)}{4(x+2)} = \frac{15x}{4x+8}$$

Example 2: Simplify $\dfrac{\frac{x}{2}+4}{3x-2}$.

$$\frac{\frac{x}{2}+4}{3x-2} = \frac{\frac{x+8}{2}}{\frac{3x-2}{1}} = \frac{(x+8)(1)}{2(3x-2)} = \frac{x+8}{6x-4}$$

Simplify each complex fraction.

1. $\dfrac{\frac{2x}{5}}{\frac{y}{6}}$

2. $\dfrac{\frac{4}{5x}}{\frac{3}{x}}$

3. $\dfrac{x-3}{\frac{2x+1}{4}}$

4. $\dfrac{x^2+\frac{1}{3}}{4x+\frac{1}{3}}$

5. $\dfrac{1-x^{-1}}{\frac{2x}{5}-1}$

6. $\dfrac{x+2x^{-2}}{2+\frac{x}{3}}$

7. $\dfrac{x}{x+\dfrac{1}{x+\frac{1}{x}}}$

8. $\dfrac{x+2}{x-2+\dfrac{1}{x+2+\frac{1}{x}}}$

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Dividing Polynomials

To divide 864 by 16, you can use long division and find that the remainder is 0. Therefore, 16 is said to be a **factor** of 864. Likewise, to divide polynomials, you can use long division. Each term may be an algebraic expression, and if the remainder is 0, the divisor is a factor of the dividend. If the divisor is not a factor of the dividend, the remainder will not be zero.

Examples: Find each quotient.

a. $(4x + 2) \div (2x + 1)$

$$\begin{array}{r} 2 \\ 2x+1\overline{)4x+2} \\ \underline{4x+2} \\ 0 \end{array}$$

$4x \div 2x = 2$
Multiply 2 and $2x + 1$.
Subtract.

Therefore, $(4x + 2) \div (2x + 1) = 2$.

b. $(x^2 - 4x + 3) \div (x - 1)$

$$\begin{array}{r} x-3 \\ x-1\overline{)x^2-4x+3} \\ \underline{x^2-x} \\ -3x+3 \\ \underline{-3x+3} \\ 0 \end{array}$$

$x^2 \div x = x$
Multiply x and $x - 1$.
Subtract; bring down 3.
Multiply -3 and $x - 1$.
Subtract.

Therefore, $(x^2 - 4x + 3) \div (x - 1) = x - 3$.

c. $(2x^2 + 9x + 8) \div (x + 3)$

$$\begin{array}{r} 2x+3 \\ x+3\overline{)2x^2+9x+8} \\ \underline{2x^2+6x} \\ 3x+8 \\ \underline{3x+9} \\ -1 \end{array}$$

$2x^2 \div x = 2x$
Multiply $2x$ and $x + 3$.
Subtract; bring down 8.
Multiply 3 and $x + 3$.
Subtract. The remainder is -1.

Therefore, $(2x^2 + 9x + 8) \div (x + 3) = 2x + 3 + \frac{-1}{x+3}$.

Find each quotient.

1. $(6x - 3) \div (2x - 1)$

2. $(x^2 - 2x + 1) \div (x - 1)$

3. $(x^2 + 5x + 4) \div (x + 4)$

4. $(2x^2 - 4x) \div (x - 2)$

5. $(5r^3 - 15r^2) \div (r - 3)$

6. $(a^2 + 6a + 5) \div (a + 5)$

7. $(2a^2 - 5a - 3) \div (2a + 1)$

8. $(6x^2 - 2x + 5) \div (x + 1)$

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Practice

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Pages 650–655

Dividing Polynomials

Find each quotient.

1. $(4x - 2) \div (2x - 1)$

2. $(y^2 + 5y) \div (y + 5)$

3. $(9a^2 + 6a) \div (3a + 2)$

4. $(8n^3 - 4n^2) \div (4n - 2)$

5. $(x^2 - 9x + 18) \div (x - 6)$

6. $(b^2 - b - 20) \div (b - 5)$

7. $(y^2 + 4y + 4) \div (y + 2)$

8. $(m^2 - 5m - 6) \div (m + 1)$

9. $(b^2 + 11b + 30) \div (b + 4)$

10. $(x^2 - 6x + 9) \div (x - 2)$

11. $(r^2 - 4) \div (r + 3)$

12. $(4x^2 + 6x + 5) \div (2x - 2)$

13. $(3n^2 - 11n + 8) \div (n - 3)$

14. $(6y^2 + 5y - 3) \div (3y + 1)$

15. $(s^3 - 1) \div (s - 1)$

16. $(a^3 + 4a + 16) \div (a + 2)$

17. $(m^3 - 9) \div (m - 2)$

18. $(x^3 - 7x - 8) \div (x + 1)$

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Enrichment

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Synthetic Division

You can divide a polynomial such as $3x^3 - 4x^2 - 3x - 2$ by a binomial such as $x - 3$ by a process called **synthetic division.** Compare the process with long division in the following explanation.

Example: Divide $(3x^3 - 4x^2 - 3x - 2)$ by $(x - 3)$ using synthetic division.

1. Show the coefficients of the terms in descending order.
2. The divisor is $x - 3$. Since 3 is to be subtracted, write 3 in the corner $\lrcorner$.
3. Bring down the first coefficient, 3.
4. Multiply. $3 \cdot 3 = 9$
5. Add. $-4 + 9 = 5$
6. Multiply. $3 \cdot 5 = 15$
7. Add. $-3 + 15 = 12$
8. Multiply. $3 \cdot 12 = 36$
9. Add. $-2 + 36 = 34$

$$\begin{array}{c|cccc} & 3 & -4 & -3 & -2 \\ & & 9 & 15 & 36 \\ \hline 3 & 3 & 5 & 12 & 34 \end{array}$$

$3x^2 + 5x + 12$, remainder 34

Check: Use long division.

$$\begin{array}{r} 3x^2 + 5x + 12 \\ x - 3\overline{)3x^3 - 4x^2 - 3x - 2} \\ \underline{3x^3 - 9x^2} \qquad\qquad \\ 5x^2 - 3x \qquad \\ \underline{5x^2 - 15x} \qquad \\ 12x - 2 \\ \underline{12x - 36} \\ 34 \end{array}$$

The result is $3x^2 + 5x + 12 + \frac{34}{x - 3}$.

Divide by using synthetic division. Check your result using long division.

1. $(x^3 + 6x^2 + 3x + 1) \div (x - 2)$

2. $(x^3 - 3x^2 - 6x - 20) \div (x - 5)$

3. $(2x^3 - 5x + 1) \div (x + 1)$

4. $(3x^3 - 7x^2 + 4) \div (x - 2)$

5. $(x^3 + 2x^2 - x + 4) \div (x + 3)$

6. $(x^3 + 4x^2 - 3x - 11) \div (x - 4)$

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Study Guide

Combining Rational Expressions with Like Denominators

You know that to add or subtract fractions with like denominators, you add or subtract the numerators and then write the sum or difference over the common denominator. For example, the sum of $\frac{1}{5}$ and $\frac{3}{5}$ is $\frac{4}{5}$. Use this same method to add or subtract rational expressions with like denominators.

Examples: Find each sum or difference. Express the answer in simplest form.

a. $\frac{a}{5} + \frac{3a}{5} = \frac{a + 3a}{5}$ *The common denominator is 5. Add the numerators.*

$= \frac{4a}{5}$

b. $\frac{8}{5m} - \frac{3}{5m} = \frac{5}{5m}$ *The common denominator is 5m. Subtract the numerators.*

$= \frac{1}{m}$ *Divide by the GCF, 5.*

c. $\frac{c}{c-1} + \frac{c-2}{c-1} = \frac{c + c - 2}{c-1}$ *The common denominator is c − 1. Add the numerators.*

$= \frac{2c-2}{c-1}$ *Factor the numerator*

$= \frac{2(c-1)}{c-1}$ *Divide by the GCF, c − 1.*

$= 2$

Find each sum or difference. Write in simplest form.

1. $\frac{6}{m} - \frac{2}{m}$

2. $\frac{4}{3x} + \frac{2}{3x}$

3. $\frac{f}{8} + \frac{7f}{8}$

4. $\frac{6t}{13} - \frac{5t}{13}$

5. $\frac{18}{25r} - \frac{3}{25r}$

6. $\frac{9x}{10} + \frac{x}{10}$

7. $\frac{8}{6n} + \frac{-2}{6n}$

8. $\frac{1}{x-2} + \frac{3}{x-2}$

9. $\frac{8}{y+1} - \frac{1}{y+1}$

10. $\frac{b}{b-1} - \frac{1}{b-1}$

11. $\frac{v}{v+2} - \frac{v}{v+2}$

12. $\frac{h}{h+4} + \frac{h+8}{h+4}$

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Practice

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Pages 656–661

Combining Rational Expressions with Like Denominators

Find each sum or difference. Write in simplest form.

1. $\frac{8}{n} + \frac{4}{n}$

2. $\frac{3x}{9} + \frac{4x}{9}$

3. $\frac{7}{2k} - \frac{5}{2k}$

4. $\frac{6n}{n} - \frac{3n}{n}$

5. $\frac{-5a}{2} + \frac{4a}{2}$

6. $\frac{2y}{3} + \frac{y}{3}$

7. $\frac{9x}{11} - \frac{8x}{11}$

8. $\frac{6p}{5} - \frac{p}{5}$

9. $\frac{9}{16q} + \frac{3}{16q}$

10. $\frac{4t}{9} - \frac{t}{9}$

11. $\frac{1}{4m} - \frac{3}{4m}$

12. $\frac{-2}{10x} + \frac{6}{10x}$

13. $\frac{6s}{7} + \frac{8s}{7}$

14. $\frac{8}{3y} - \frac{2}{3y}$

15. $\frac{4}{x-7} - \frac{2}{x-7}$

16. $\frac{-2}{x+3} + \frac{3}{x+3}$

17. $\frac{5}{y-4} - \frac{8}{y-4}$

18. $\frac{3m}{m+2} - \frac{m}{m+2}$

19. $\frac{3n}{n-1} + \frac{2}{n-1}$

20. $\frac{5a}{a+4} - \frac{7}{a+4}$

21. $\frac{4g}{g+3} + \frac{12}{g+3}$

22. $\frac{2r+2}{r-5} - \frac{r-4}{r-5}$

23. $\frac{s-3}{s+1} + \frac{4s+8}{s+1}$

24. $\frac{-11b}{5b+3} + \frac{12b-2}{5b+3}$

25. $\frac{15y}{4y-2} - \frac{3y+6}{4y-2}$

26. $\frac{5c+3}{2c+1} + \frac{9c+4}{2c+1}$

27. $\frac{2x+3}{3x+4} - \frac{8x+11}{3x+4}$

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Enrichment

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Sum and Difference of Any Two Like Powers

The sum of any two like powers can be written $a^n + b^n$, where n is a positive integer. The difference of like powers is $a^n - b^n$. Under what conditions are these expressions exactly divisible by $(a + b)$ or $(a - b)$? The answer depends on whether n is an odd or even number.

Use long division to find the following quotients. (HINT: Write $a^3 + b^3$ as $a^3 + 0a^2 + 0a + b^3$.) Is the numerator exactly divisible by the denominator? Write yes or no.

1. $\frac{a^3 + b^3}{a + b}$

2. $\frac{a^3 + b^3}{a - b}$

3. $\frac{a^3 - b^3}{a + b}$

4. $\frac{a^3 - b^3}{a - b}$

5. $\frac{a^4 + b^4}{a + b}$

6. $\frac{a^4 + b^4}{a - b}$

7. $\frac{a^4 - b^4}{a + b}$

8. $\frac{a^4 - b^4}{a - b}$

9. $\frac{a^5 + b^5}{a + b}$

10. $\frac{a^5 + b^5}{a - b}$

11. $\frac{a^5 - b^5}{a + b}$

12. $\frac{a^5 - b^5}{a - b}$

13. Use the words *odd* and *even* to complete these two statements.

a. $a^n + b^n$ is divisible by $a + b$ if n is ______, and by neither $a + b$ nor $a - b$ if n is ______.

b. $a^n - b^n$ is divisible by $a - b$ if n is ______, and by both $a + b$ and $a - b$ if n is ______.

14. Describe the signs of the terms of the quotients when the divisor is $a - b$.

15. Describe the signs of the terms of the quotient when the divisor is $a + b$.

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Combining Rational Expressions with Unlike Denominators

You add $\frac{1}{5}$ and $\frac{3}{4}$ by first finding the common denominator, 20.

Likewise, to add or subtract rational expressions with unlike denominators, first rename the expressions so the denominators are alike. Then add or subtract the numerators and write the sum or difference over the common denominator. Simplify if necessary. The least common denominator (LCD) may make the computations easier.

Example: Find $\frac{3}{4a^2} + \frac{5}{2a}$.

Step 1 First find the LCD.

$4a^2 = 2 \cdot 2 \cdot a \cdot a$

$2a = 2 \cdot a$

The LCD is $4a^2$.

Step 2 Rename each expression with the LCD as denominator.

The denominator of $\frac{3}{4a^2}$ is already $4a^2$, so only $\frac{5}{2a}$ needs to be renamed.

$\frac{5}{2a} = \frac{5}{2a} \cdot \frac{2a}{2a} = \frac{10a}{4a^2}$

Step 3 Add.

$\frac{3}{4a^2} + \frac{5}{2a} = \frac{3}{4a^2} + \frac{10a}{4a^2}$

$= \frac{3 + 10a}{4a^2}$ *The expression is in simplest form.*

Find each sum or difference. Write in simplest form.

1. $\frac{x}{6} + \frac{x}{12}$

2. $\frac{f}{8} - \frac{f}{16}$

3. $\frac{d}{6} + \frac{d}{3}$

4. $\frac{4}{x} - \frac{6}{2x}$

5. $\frac{3d}{4} - \frac{d}{8}$

6. $\frac{3x}{m} - \frac{1}{2m}$

7. $\frac{3}{m} - \frac{4}{m^2}$

8. $\frac{6}{x} + \frac{4}{x^3}$

9. $\frac{3}{4y^2} + \frac{1}{8y}$

10. $\frac{1}{x} + \frac{1}{y}$

11. $\frac{2c}{a} - \frac{4}{b^2}$

12. $\frac{3}{2n} + \frac{2t}{4n^2}$

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Practice

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Pages 662–667

Combining Rational Expressions with Unlike Denominators

Find the LCM for each pair of expressions.

1. $4ab$, $18b$

2. $6x^2y$, $9xy$

3. $10a^2$, $12ab^2$

4. $y + 2$, $y^2 - 4$

5. $x^2 - 9$, $x^2 + 5x + 6$

6. $x^2 - 3x - 4$, $2x^2 - 2x - 8$

Write each pair of expressions with the same LCD.

7. $\frac{4}{b}, \frac{5}{ab}$

8. $\frac{5}{6c^2}, \frac{3}{8c}$

9. $\frac{6}{7x^2y}, \frac{4}{5xy}$

10. $\frac{3}{r + 4}, \frac{7}{2r + 8}$

11. $\frac{x}{x - 2}, \frac{x + 1}{x - 5}$

12. $\frac{3}{y - 4}, \frac{2y}{y^2 - 16}$

Find each sum or difference. Write in simplest form.

13. $\frac{2k}{8} + \frac{3k}{16}$

14. $\frac{n}{2} - \frac{n}{7}$

15. $\frac{7}{3b} - \frac{3}{b}$

16. $\frac{5}{x} + \frac{3}{y}$

17. $\frac{2}{m^2n} - \frac{6}{mn}$

18. $\frac{c}{4c} + \frac{8}{c}$

19. $\frac{1}{6a} - \frac{2}{9a^2}$

20. $\frac{2}{3ab} + \frac{3b}{4ab}$

21. $\frac{p}{4p^2q} + \frac{3}{5pq}$

22. $\frac{2x}{3xy^2} - \frac{2}{5xy}$

23. $\frac{2s}{s^2 - 4} + \frac{4}{s + 2}$

24. $\frac{b}{b^2 - 9} - \frac{5}{b - 3}$

25. $\frac{-5}{2r + 3} + \frac{7}{6r + 9}$

26. $\frac{6}{y + 2} + \frac{3}{y}$

27. $\frac{x}{x - 3} - \frac{2}{x - 4}$

Enrichment

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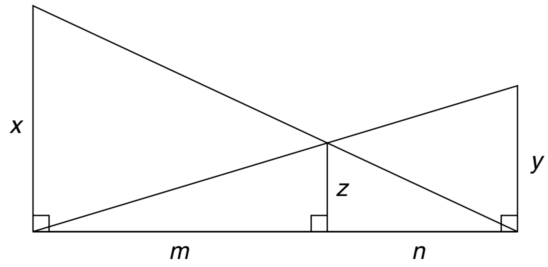
Work Problems and Similar Right Triangles



"The work problem" has been included in algebra textbooks for a very long time. In older books, the people in the problem always seemed to be digging ditches.

> If Olivia can dig a ditch in x hours and George can dig the same ditch in y hours, how long will it take them to dig the ditch if they work together?

You have learned a way to solve this type of problem using rational equations. It can also be solved using a geometric model that uses two overlapping right triangles.

In the drawing, the length x is Olivia's time. The length y is George's time. The answer to the problem is the length of the segment z. The distance $m + n$ can be any convenient length.

Solve each problem.

1. Solve the work problem for $x = 6$ and $y = 3$ by drawing a diagram and measuring.

2. Confirm your solution to problem 1 by writing and solving a rational equation.

3. On a separate sheet of paper, create a word problem to go with the values $x = 6$ and $y = 3$.

4. On a separate sheet of paper, solve this problem with a diagram. Use centimeters and measure to the nearest tenth. Olivia can wash a car in 3 hours. George can wash a car in 4 hours. How long will it take them working together to wash one car?

5. Triangles that have the same shape are called **similar triangles**. You may have learned that corresponding sides of similar triangles form equal ratios. Using the drawing at the top of the page, you can thus conclude that Equations A and B below are true. Use the equations to prove the formula for the work problem.

Equation A

$$\frac{z}{x} = \frac{n}{m + n}$$

Equation B

$$\frac{z}{y} = \frac{m}{m + n}$$

Work Formula

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

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Study Guide

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Solving Rational Equations

A rational equation is an equation that contains at least one rational expression. There are three steps in solving rational equations.

Step 1 Find the LCD of all terms.

Step 2 Multiply each side of the equation by the LCD.

Step 3 Use the Distributive Property to simplify.

Example: Solve $\frac{3}{5x} + \frac{2}{x} = \frac{1}{5}$.

$$\frac{3}{5x} + \frac{2}{x} = \frac{1}{5} \qquad \textit{The LCD is } 5x.$$

$$5x\left(\frac{3}{5x} + \frac{2}{x}\right) = 5x\left(\frac{1}{5}\right) \qquad \textit{Multiply each side by the LCD.}$$

$$5x\left(\frac{3}{5x}\right) + 5x\left(\frac{2}{x}\right) = 5x\left(\frac{1}{5}\right) \qquad \textit{Distributive Property}$$

$$\overset{1}{\cancel{5x}}\left(\frac{3}{\underset{1}{\cancel{5x}}}\right) + \overset{5}{5\cancel{x}}\left(\frac{2}{\underset{1}{\cancel{x}}}\right) = \overset{x}{\cancel{5}x}\left(\frac{1}{\underset{1}{\cancel{5}}}\right)$$

$$3 + 10 = x \qquad \textit{Simplify.}$$

$$13 = x$$

Check:

$$\frac{3}{5x} + \frac{2}{x} = \frac{1}{5}$$

$$\frac{3}{5(13)} + \frac{2}{13} \stackrel{?}{=} \frac{1}{5}$$

$$\frac{3}{65} + \frac{2}{13} \stackrel{?}{=} \frac{1}{5} \qquad \textit{The LCD is } 65.$$

$$\frac{3}{65} + \frac{10}{65} \stackrel{?}{=} \frac{13}{65}$$

$$\frac{13}{65} = \frac{13}{65} \checkmark$$

Solve each equation. Check your solution.

1. $\frac{x}{6} + \frac{x}{12} = \frac{1}{2}$

2. $\frac{3x}{8} - \frac{x}{4} = \frac{1}{4}$

3. $\frac{y}{3} + \frac{2y}{5} = \frac{11}{3}$

4. $\frac{9y}{10} - \frac{y}{2} = \frac{2}{5}$

5. $\frac{3d}{4} - \frac{d}{8} = \frac{5}{16}$

6. $\frac{x}{4} = \frac{x + 2}{8}$

7. $\frac{t - 4}{6} = \frac{t + 1}{8}$

8. $\frac{x + 1}{x} + \frac{x - 2}{x} = 4$

9. $\frac{8}{4 - s} - \frac{s}{4 - s} = 2$

10. $\frac{x}{8} - \frac{x}{4} = \frac{x - 1}{2}$

11. $\frac{m + 2}{m} - \frac{m - 1}{m} = 3$

12. $\frac{1}{4s} - \frac{3}{2s} = \frac{1}{8}$

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Practice

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Solving Rational Equations

Solve each equation. Check your solution.

1. $\frac{c}{2} + \frac{c}{2} = \frac{1}{2}$

2. $\frac{3b}{5} - \frac{1}{5} = \frac{b}{5}$

3. $\frac{8}{a} = \frac{12}{a} + 5$

4. $\frac{7}{b} - 2 = \frac{3}{b}$

5. $\frac{7}{9t} - \frac{5}{6t} = \frac{1}{3}$

6. $\frac{4}{5x} + \frac{1}{4x} = \frac{3}{4}$

7. $\frac{3x}{4} - \frac{2x}{3} = \frac{1}{4}$

8. $\frac{s + 7}{6} - 2 = \frac{s}{4}$

9. $\frac{n - 3}{2} = \frac{n}{5} + 3$

10. $\frac{y + 6}{3} - \frac{y + 12}{7} = 2$

11. $\frac{11}{p - 2} - \frac{2}{p - 2} = -8$

12. $\frac{x + 5}{2x} + \frac{x + 3}{3x} = \frac{1}{3}$

13. $\frac{2}{n} - \frac{3}{n + 1} = \frac{3}{n + 1}$

14. $\frac{6}{y - 3} - \frac{5}{y} = \frac{3}{y}$

15. $\frac{6}{s} + \frac{3s}{s - 2} - 2 = 3$

16. $\frac{5}{k} + \frac{k - 2}{k + 1} = 1$

17. $\frac{r + 2}{r} - \frac{r + 2}{r - 5} = -\frac{3}{r - 5}$

18. $\frac{2c}{c + 3} - \frac{4}{2c + 6} = 4$

19. $\frac{3b}{b + 2} + \frac{3}{3b + 6} = 2$

20. $\frac{2m}{m + 3} - \frac{4}{m - 3} = 2$

21. $\frac{y}{y + 2} + \frac{3}{y - 2} = \frac{y}{y - 2}$

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Enrichment

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Using Rational Expressions and Equations

In 1985 Steve Cram set a world record for the mile run with a time of 3:46.31. In 1954, Roger Bannister ran the first mile under 4 minutes at 3:59.4. Had they run those times in the same race, how far in front of Bannister would Cram have been at the finish?

Use $\frac{d}{t} = r$. Since 3 min 46.31 s = 226.31 s, and 3 min 59.4 s = 239.4 s, Cram's rate was $\frac{5280 \text{ ft}}{226.31 \text{ s}}$ and Bannister's rate was $\frac{5280 \text{ ft}}{239.4 \text{ s}}$.

	r	*t*	*d*
Cram	$\frac{5280}{226.31}$	226.31	5280 feet
Bannister	$\frac{5280}{239.4}$	226.31	$\frac{5280}{239.4} \cdot 226.31$ or 4491.3 feet

Therefore, when Cram hit the tape, he would be 5280 − 4491.3, or 288.7 feet, ahead of Bannister. Let's see whether we can develop a formula for this type of problem.

Let D = the distance raced,
W = the winner's time,
and L = the loser's time.

Following the same pattern, you obtain the results shown in the table at the right.

	r	*t*	*d*
Winner	$\frac{D}{W}$	W	$\frac{D}{W} \cdot W = D$
Loser	$\frac{D}{L}$	W	$\frac{D}{L} \cdot W = \frac{DW}{L}$

The winning distance will be $D - \frac{DW}{L}$.

1. Show that the expression for the winning distance is equivalent to $\frac{D(L - W)}{L}$.

Use the formula winning distance = $\frac{D(L - W)}{L}$ to find the winning distance for each of the following Olympic races.

2. women's 400 meter relay: Canada 48.4 s (1928); East Germany 41.6 s (1980)

3. men's 200 meter freestyle swimming: Mark Spitz 1 min 52.78 s (1972); Michael Gross 1 min 47.44 s (1984)

4. men's 50,000 meter walk: Thomas Green 4 h 50 min 10 s (1932); Hartwig Gauter 3 h 49 min 24 s (1980)

5. women's 400 meter freestyle relay: Great Britain 5 min 52.8 s (1912); East Germany 3 min 42.71 s (1980)

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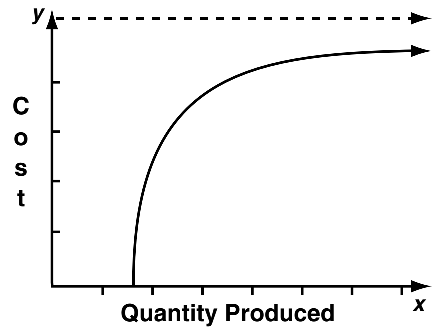
School-to-Workplace

Production Costs (Production Analyst)

Everyone likes a bargain. In fact, when goods are very expensive, people are less likely to buy them. To make a profit and to make goods available in large quantities, a manufacturer must conduct studies that provide ways to keep costs down.



The graph at the right shows how production costs can level off when very large quantities are produced. The cost of production increases, of course, but its rate of increase slows down.

Suppose a production analyst finds that the production cost for making x units of product A is given by the expression below.

$$2000 - \frac{2000}{2x - 3}$$

On the assembly line for product B, the analyst finds that the cost for producing y units is given by the expression below.

$$3500 - \frac{3500}{4y - 5}$$

Find an expression for the cost of making products A and B.

$$\left(2000 - \frac{2000}{2x - 3}\right) + \left(3500 - \frac{3500}{4y - 5}\right) = 5500 - \frac{4y - 5}{4y - 5} \cdot \frac{2000}{2x - 3} - \frac{2x - 3}{2x - 3} \cdot \frac{3500}{4y - 5}$$

$$= 5500 - \frac{8000y - 10{,}000 + 7000x - 10{,}500}{(2x - 3)(4y - 5)}$$

$$= 5500 - \frac{8000y + 7000x - 20{,}500}{8xy - 12y - 10x + 15}$$

Solve.

1. A certain product consists of 2 parts (R and S) that are manufactured separately. Find the total cost of producing x units of the product if the production costs for the parts are given by the expressions below.

R: $3000 - \frac{3000}{x - 1}$ S: $2000 - \frac{2000}{x + 1}$

2. Find the value of the expression in Exercise 1 if $x = 500$.

3. Find the value of the expression in the example if $x = 500$ and $y = 500$.