

Study Guide

Solving Proportions

An equation stating that two ratios are equal is called a **proportion**.

For example, $\frac{8}{3} = \frac{16}{6}$ is a proportion. Use the Property of

Proportions and other algebraic properties you know to solve proportions.

Property of Proportions

The cross products of a proportion are equal.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$.

Example 1: Solve $\frac{6}{5} = \frac{18}{n}$.

$$\frac{6}{5} = \frac{18}{n}$$

$$6n = 5(18) \quad \text{Find the cross products.}$$

$$6n = 90 \quad \text{Simplify.}$$

$$n = 15$$

Check: $\frac{6}{5} \stackrel{?}{=} \frac{18}{15}$
 $6(15) \stackrel{?}{=} 5(18)$
 $90 = 90 \checkmark$

Example 2: Solve $\frac{p}{12} = \frac{p-4}{6}$.

$$\frac{p}{12} = \frac{p-4}{6}$$

$$6p = 12(p-4)$$

$$6p = 12p - 48$$

$$-6p = -48$$

$$p = 8$$

Check: $\frac{8}{12} \stackrel{?}{=} \frac{8-4}{6}$
 $8(6) \stackrel{?}{=} 12(8-4)$
 $48 = 48 \checkmark$

Solve each proportion.

1. $\frac{g}{3} = \frac{12}{4}$

2. $\frac{7}{4} = \frac{35}{m}$

3. $\frac{2}{3} = \frac{a}{60}$

4. $\frac{5}{8} = \frac{x}{60}$

5. $\frac{5}{b} = \frac{65}{39}$

6. $\frac{c}{38} = \frac{2}{4}$

7. $\frac{d}{2} = \frac{2}{8}$

8. $\frac{k}{10} = \frac{1}{1000}$

9. $\frac{8}{x-2} = \frac{24}{7}$

10. $\frac{y}{9} = \frac{y+2}{3}$

11. $\frac{x}{x-3} = \frac{18}{6}$

12. $\frac{5}{x-2} = \frac{8}{x+1}$

Practice**Solving Proportions***Solve each proportion.*

1. $\frac{1}{7} = \frac{a}{14}$

2. $\frac{8}{2} = \frac{12}{h}$

3. $\frac{5}{b} = \frac{3}{9}$

4. $\frac{9}{6} = \frac{15}{b}$

5. $\frac{3}{y} = \frac{9}{21}$

6. $\frac{21}{24} = \frac{c}{8}$

7. $\frac{8}{3} = \frac{32}{x}$

8. $\frac{4}{5} = \frac{p}{25}$

9. $\frac{24}{g} = \frac{3}{8}$

10. $\frac{5}{2} = \frac{13}{a}$

11. $\frac{1.5}{c} = \frac{15}{5}$

12. $\frac{6}{2.4} = \frac{x}{4}$

13. $\frac{4}{2} = \frac{x+2}{3}$

14. $\frac{c+5}{15} = \frac{8}{6}$

15. $\frac{7}{7} = \frac{10}{b+3}$

16. $\frac{y}{y+3} = \frac{2}{5}$

17. $\frac{z+6}{12} = \frac{z}{4}$

18. $\frac{5}{4} = \frac{a+4}{2a}$

Convert each measurement as indicated.

19. 5 pounds to ounces

20. 3000 grams to kilograms

21. 7 feet to inches

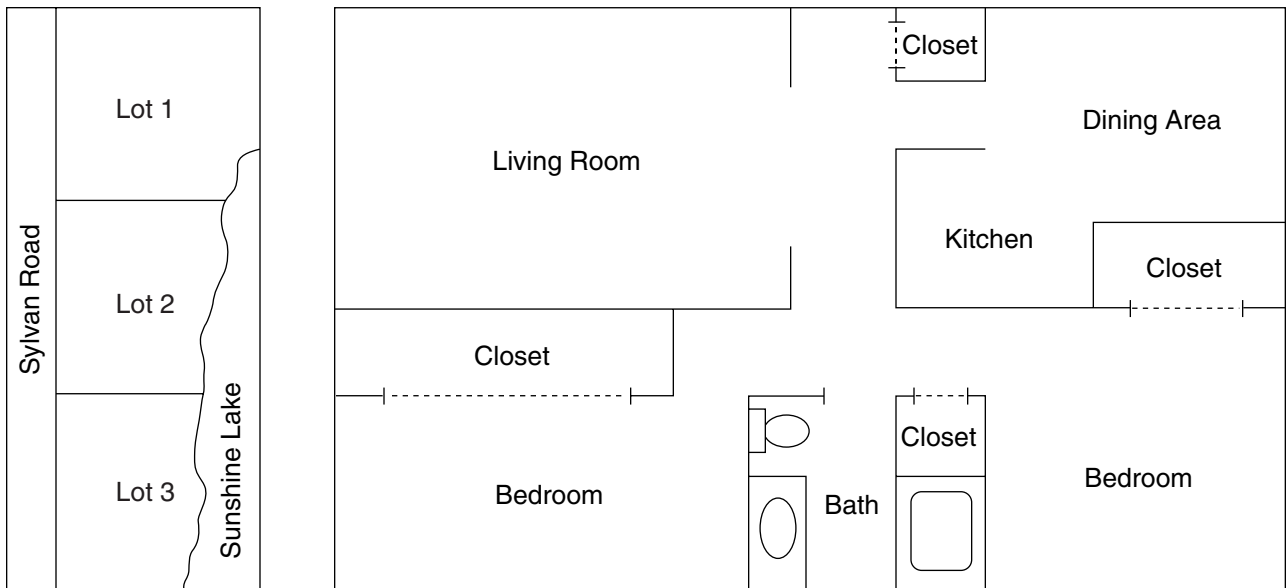
22. 4 meters to centimeters

23. 6 quarts to gallons

24. 250 centimeters to meters

Scale Drawings

The map at the left below shows building lots for sale. The scale ratio is 1:2400. At the right below is the floor plan for a two-bedroom apartment. The length of the living room is 6 m. On the plan the living room is 6 cm long.

**Answer each question.**

1. On the map, how many feet are represented by an inch?
2. On the map, measure the frontage of Lot 2 on Sylvan Road in inches. What is the actual frontage in feet?
3. What is the scale ratio represented on the floor plan?
4. On the floor plan, measure the width of the living room in centimeters. What is the actual width in meters?
5. About how many square meters of carpeting would be needed to carpet the living room?
6. Make a scale drawing of your classroom using an appropriate scale.
7. On the scale for a map of Lancaster, Pennsylvania, 2.5 cm equals 3 km. Find the scale ratio.

Scale Drawings and Models

A scale drawing or scale model is used to represent an object that is too large or too small to be drawn or built at its actual size.

Example 1: The Statue of Liberty is about 150 feet tall. A model is 15 inches tall. Find the scale used.

The scale used is the ratio of the length of the model to the actual length, or 15 in. = 150 feet. Reduce the ratio to get a scale of 1 in. = 10 feet.

Another method is to convert 150 feet to inches, or 150 ft = 150(12) in. = 1800 in. The scale is 15 in. = 1800 in. Reduce the ratio to get 1:120. The units do not need to be included if they are the same.

Example 2: The scale of a map of Florida is 1 inch = 20 miles. Find the actual distance between Miami and St. Petersburg if the distance between them on the map is 13 inches.

$$\frac{1 \text{ inch}}{20 \text{ miles}} = \frac{13 \text{ inches}}{x \text{ miles}} \quad \text{Use a proportion.}$$

$$1 \cdot x = 20(13)$$

$$x = 260$$

The distance from Miami to St. Petersburg is 260 miles.

Find each scale or distance.

- On the blueprint of a house, the kitchen is 5 inches long. If the actual kitchen is 20 feet long, find the scale of the blueprint.
- On a map, the scale is 1 inch = 25 miles. Find the actual distance for each map distance.

From	To	Map Distance
Portsmouth, OH	Springfield, OH	5.2 inches
Chicago, IL	Lawrenceville, IL	10 inches
Santa Fe, NM	Clovis, NM	$8\frac{2}{5}$ inches

- The Sears Tower is 1450 feet tall. If a model is 25 inches tall, find the scale.
- In an HO scale model of a train, the length of the engine is 6 inches. If the HO scale is 1 : 87, find the actual length. Write the answer in feet.
- Las Vegas, Nevada, is 445 miles from Reno, Nevada. If the distance on the map is $11\frac{1}{8}$ inches, find the scale used for the map.

Scale Drawings and Models

On a map, the scale is 1 inch = 30 miles. Find the actual distance for each map distance.

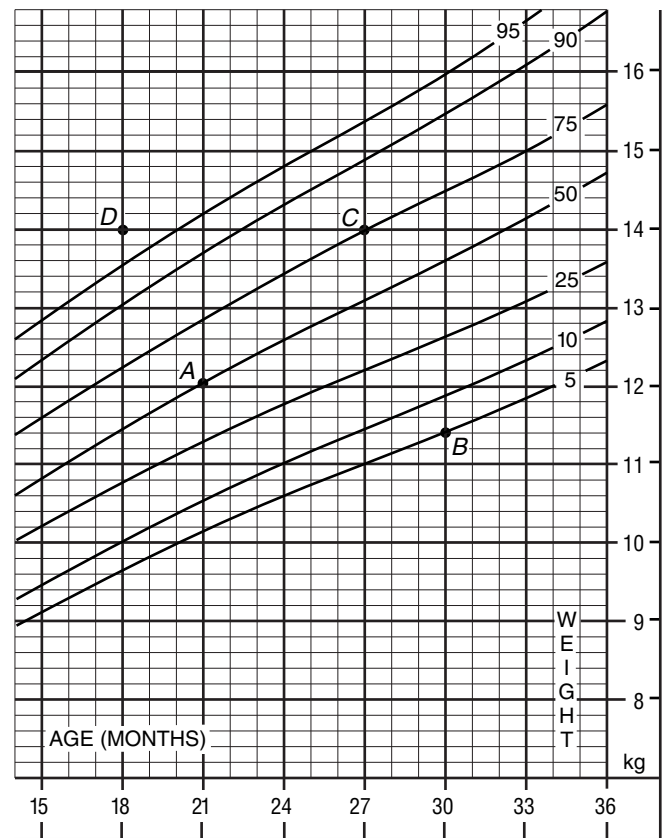
1. Los Angeles, CA, to San Bernardino, CA; 2 inches
2. Kalamazoo, MI, to Chicago, IL; 4.5 inches
3. Nashville, TN, to Union City, TN; 6 inches
4. Springfield, MO, to Joplin, MO; 2.5 inches
5. Albuquerque, NM, to Santa Fe, NM; $1\frac{3}{4}$ inches
6. Montgomery, AL, to Birmingham, AL; 3 inches
7. Columbus, OH, to Cincinnati, OH; 3.5 inches
8. Des Moines, IA, to Sioux City, IA; $6\frac{3}{4}$ inches
9. Concord, NH, to Boston, MA; $2\frac{1}{4}$ inches
10. Providence, RI, to Newport, RI; 1 inch
11. Raleigh, NC, to Wilmington, NC; 4 inches
12. St. Paul, MN, to Minneapolis, MN; $\frac{1}{4}$ inch
13. Portland, OR, to Seattle, WA; $5\frac{3}{4}$ inches

Growth Charts

Graphs are often used by doctors to show parents the growth rates of their children. The horizontal scale of the chart at the right shows the ages from 15 to 36 months. The vertical scale shows weight in kilograms. One kilogram is about 2.2 pounds. The curved lines are used to show how a child's weight compares with others of his or her age.

Look at the point labeled *A*. It represents a 21-month-old who weighs 12 kilograms. It is located on the slanted line labeled 50. This means the child's weight is in the "50th percentile." In other words, 50% of all 21-month-olds weigh more than 12 kilograms and 50% weigh less than 12 kilograms.

The location of Point *B* indicates that a 30-month-old who weighs 11.4 kilograms is in the 5th percentile. Only 5% of 30-month-old children will weigh less than 11.4 kilograms.



Solve.

- Look at the point labeled *C*. How much does the child weigh? How old is he? What percent of children his age will weigh more than he does?
- Look at the point labeled *D*. What is the child's age and weight? What percent of children her age will weigh more than she does?
- What is the 50th percentile weight for a child 27 months old?
- How much weight would child *B* have to gain to be in the 50th percentile?
- If child *D* did not gain any weight for four months, what percentile would he be in?
- How much heavier is a $2\frac{1}{2}$ -year-old in the 90th percentile than one in the 10th percentile?

Study Guide**The Percent Proportion**

Percent is a ratio that compares a number to 100. For example, 6 out of 100 can be expressed as $\frac{6}{100}$, 6:100, or 6%. Use the percent proportion to find percents.

If P is the percentage, B is the base, and r is the percent, the percent proportion is $\frac{P}{B} = \frac{r}{100}$.

Example 1: 80% of what number is 336?

$$\begin{aligned}\frac{P}{B} &= \frac{r}{100} \\ \frac{336}{B} &= \frac{80}{100} \\ 336(100) &= 80B \\ 33600 &= 80B \\ \frac{33,600}{80} &= \frac{80B}{80} \\ 420 &= B\end{aligned}$$

So, 80% of 420 = 336.

Example 2: 75 is what percent of 300?

$$\begin{aligned}\frac{P}{B} &= \frac{r}{100} \\ \frac{75}{300} &= \frac{r}{100} \\ 75(100) &= 300r \\ 7500 &= 300r \\ \frac{7500}{300} &= \frac{300r}{300} \\ 25 &= r\end{aligned}$$

So, 75 is 25% of 300.

Use the percent proportion to find each number.

1. 40 is what percent of 80?
2. 8 is what percent of 25?
3. Find 10% of 170.
4. 18 is 12% of what number?
5. What number is 150% of 12?
6. 32 is 400% of what number?
7. 30 is what percent of 150?
8. Find 84% of 500.
9. 9% of what number is 8.1?
10. What number is 15% of 42?
11. 50% of 0.75 is what number?
12. Find 140% of 82.
13. 60% of what number is 2.1?
14. Find 0.5% of 8.
15. 600 is what percent of 240?
16. 1 is what percent of 250?

Practice***The Percent Proportion***

Express each fraction or ratio as a percent.

1. $\frac{3}{4}$

2. 4 out of 5

3. 4 to 10

4. 7 to 4

5. $\frac{13}{20}$

6. 1 out of 8

7. $\frac{1}{5}$

8. 2 out of 4

9. 6 to 5

10. Two out of 50 students scored above 98 on a geometry test.

11. At a computer convention, 19 out of 20 people accepted a free mouse pad.

12. In a recent inspection, three-eighths of the apartments at Kendall Heights had fire extinguishers.

Use the percent proportion to find each number.

13. 20 is what percent of 125?

14. Find 30% of 75.

15. 18 is 45% of what number?

16. 85% of what number is 85?

17. 15 is what percent of 50?

18. What number is 3% of 40?

19. 40% of what number is 28?

20. Find 130% of 20.

21. 78 is 65% of what number?

22. What is 10% of 73?

23. 30 is what percent of 150?

24. Find 6% of 15.

Partnerships

Two or more people who agree to combine their money, goods, or skill in some business often do so by forming a *partnership*. Gains and losses in a partnership are usually divided in proportion to the amount of money each person has invested.

Sharon and Tina start a business to print restaurant menus using Tina's computer. For supplies and software, Sharon invests \$1200 and Tina, \$800. If their profit after the first year is \$500, what is each person's share?

The entire investment is \$1200 plus \$800, or \$2000.

Sharon's share is $\frac{\$1200}{\$2238}$, or $\frac{3}{5}$. Tina's share is $\frac{\$800}{\$2238}$, or $\frac{2}{5}$.

Answer each question.

- In the problem above, what is Sharon's share of the profit?
- What is Tina's share?
- Sal and Mario were partners in a construction business. Sal put in \$5238 and Mario, \$4238. Their profit in 3 years was \$4500. What was each partner's share?
- Tim, Bob, and Alice entered into a business partnership for 2 years. Tim put in \$3600, Bob put in \$2400, and Alice put in \$2238. If their profit was \$3238, what was each person's share?
- Fred, Vito, and Hal began with a capital of \$28,238. Fred furnished \$7238; Vito, \$6238; and Hal, the remainder. If they gained 14% on their investment, what was each person's share?
- A storeroom belonging to Taylor, Bartinelli, & Wong was entirely destroyed by fire. They received \$9675 insurance. What was each person's share, if Taylor owned 15%, Bartinelli, 40%, and Wong the remainder?
- Divide a profit of \$7200 between two partners so that the ratio of the shares is 4 to 5.
- Divide a profit of \$18,238 among three partners so that the ratio of the shares is 1:2:3.

Study Guide**The Percent Equation**

The percent proportion $\frac{P}{B} = \frac{r}{100}$ can also be written as the equation

$P = RB$, where P is the percentage, B is the base, and R is the rate expressed as a decimal.

Example 1: Find 60% of 225.

$$P = RB$$

$$R = 0.6 \quad \text{Write } R \text{ as decimal.}$$

$$P = 0.6(225)$$

$$P = 135$$

So, 60% of 225 = 135.

Example 2: 18 is 40% of what number?

$$P = RB$$

$$R = 0.4$$

$$18 = 0.4(B)$$

$$\frac{18}{0.4} = \frac{0.4B}{0.4} \quad \text{Divide by } 0.4.$$

$$45 = B$$

So, 18 is 40% of 45.

Use the percent equation to find each number.

- 72 is what percent of 90?
- Find 10% of 240.
- 90 is what percent of 360?
- Find 8% of 120.
- Find 32% of 600.
- 14% of what number is 9.8?
- What number is 24% of 90?
- 150% of 0.6 is what number?
- 750 is what percent of 150?
- 48 is 300% of what number?
- 2 is what percent of 500?
- Find 0.5% of 12.
- Find 160% of 85.
- What number is 265% of 24?
- 150 is what percent of 25?
- 98 is 5% of what number?
- Marcia bought \$80 in groceries. What is her final cost after sales tax of 6.5% is added?
- Mr. Perez paid 14% of his income in taxes. If his taxes were \$7686, what was his income?

The Percent Equation

Use the percent equation to find each number.

1. Find 60% of 150.
2. What number is 40% of 95?
3. 21 is 70% of what number?
4. Find 20% of 120.
5. Find 7% of 80.
6. 63 is 60% of what number?
7. 12 is 30% of what number?
8. 90 is 45% of what number?
9. What number is 27% of 50?
10. What number is 70% of 122?
11. What number is 12% of 85?
12. Find 14% of 150.
13. 26 is 65% of what number?
14. What number is 67% of 140?
15. 108 is 90% of what number?
16. Find 34% of 85.
17. 50 is 25% of what number?
18. What number is 95% of 90?
19. 21 is 35% of what number?
20. Find 22% of 55.
21. Find 14.5% of 500.
22. 4 is 0.8% of what number?

Using Percent

Use what you have learned about percent to solve each problem.

A TV movie had a “rating” of 15 and a 25 “share.” The rating of 15 means that 15% of the nation’s total TV households were tuned in to this show. The 25 share means that 25% of the homes with TVs turned on were tuned to the movie. How many TV households had their TVs turned off at this time?

To find out, let T = the number of TV households
and x = the number of TV households with the TV off.
Then $T - x$ = the number of TV households with the TV on.

Since $0.15T$ and $0.25(T - x)$ both represent the number of households tuned to the movie,

$$0.15T = 0.25(T - x)$$

$$0.15T = 0.25T - 0.25x.$$

Solve for x .

$$0.25x = 0.10T$$

$$x = \frac{0.10T}{0.25} = 0.40T$$

Forty percent of the TV households had their TVs off when the movie was aired.

Answer each question.

1. During that same week, a sports broadcast had a rating of 22.1 and a 43 share. Show that the percent of TV households with their TVs off was about 48.6%.
2. Find the percent of TV households with their TVs turned off during a show with a rating of 18.9 and a 29 share.
3. Show that if T is the number of TV households, r is the rating, and s is the share, then the number of TV households with the TV off is $\frac{(s - r)T}{s}$.
4. If the fraction of TV households with no TV on is $\frac{s - r}{s}$ then show that the fraction of TV households with TVs on is $\frac{r}{s}$.
5. Find the percent of TV households with TVs on during the most watched serial program in history: the last episode of *M*A*S*H*, which had a 60.3 rating and a 77 share.
6. A local station now has a 2 share. Each share is worth \$50,000 in advertising revenue per month. The station is thinking of going commercial free for the three months of summer to gain more listeners. What would its new share have to be for the last 4 months of the year to make more money for the year than it would have made had it not gone commercial free?

Study Guide**Percent of Change**

The median price of a home increased from \$90,600 in 1988 to \$134,600 in 1998. The change in the price can be written as a percent. Since the price increased, it is called the **percent of increase**. Likewise, the percent that an amount decreases is known as the **percent of decrease**.

To find the percent of increase or decrease, write a ratio that compares the amount of increase or decrease to the original amount.

Example: Find the percent of increase in the median price of a home.

original price: \$90,600 new price: \$134,600

The amount of increase = $134,600 - 90,600 = 44,000$. Substitute the amount of increase for P . You are comparing it to the original price, so substitute \$90,600 for B .

Method 1

$$\begin{aligned}\frac{P}{B} &= \frac{r}{100} \\ \frac{44,000}{90,600} &= \frac{r}{100} \\ 44,000(100) &= 90,600r \\ 4,400,000 &= 90,600r \\ \frac{4,400,000}{90,600} &= \frac{90,600r}{90,600} \\ 48.6 &= r\end{aligned}$$

Method 2

$$\begin{aligned}P &= RB \\ 44,000 &= R \cdot 90,600 \\ \frac{44,000}{90,600} &= R \cdot \frac{90,600}{90,600} && \text{Divide each side} \\ 0.486 &= R && \text{by } 90,600.\end{aligned}$$

So the percent of increase was about 49%.

Find the percent of change. Round to the nearest percent.

- original: 25
new: 26
- original: \$20,000
new: \$17,500
- original: 96
new: 54
- original: 15
new: 33
- In January, gasoline cost \$1.05 per gallon. In July, gasoline cost \$1.20 per gallon. Find the percent of increase.
- The original price of a CD player is \$105. A discount of \$21 is given. Find the percent of decrease.
- The original cost of a computer is \$900. If the price is decreased by 15%, what is the sale price?

Practice***Percent of Change***

Find the percent of increase or decrease. Round to the nearest percent.

1. original: 60
new: 54
2. original: 20
new: 25
3. original: 18
new: 36
4. original: 50
new: 32
5. original: 32
new: 20
6. original: 35
new: 98

The cost of an item and a sales tax rate are given. Find the total price of each item to the nearest cent.

7. guitar: \$120; 5%
8. shirt: \$22.95; 6%
9. shoes: \$49.99; 7%
10. jacket: \$89.95; 6%
11. ruler: \$1.49; 5%
12. weight bench: \$79; 6%

The original cost of an item and a discount rate are given. Find the sale price of each item to the nearest cent.

13. stereo: \$900; 10%
14. jeans: \$54; 25%
15. VCR: \$129.95; 20%
16. golf club: \$69.95; 15%
17. barrette: \$6.99; 15%
18. sweat pants: \$12; 25%

Compound Interest

In most banks, interest on savings accounts is compounded at set time periods such as three or six months. At the end of each period, the bank adds the interest earned to the account.

During the next period, the bank pays interest on all the money in the bank, including interest. In this way, the account earns interest on interest.

Suppose Ms. Tanner has \$1000 in an account that is compounded quarterly at 5%. Find the balance after the first two quarters.

Use $I = prt$ to find the interest earned in the first quarter if $p = 1000$ and $r = 5\%$. Why is t equal to $\frac{1}{4}$?

$$\begin{aligned} \text{First quarter: } I &= 1000 \times 0.05 \times \frac{1}{4} \\ I &= 12.50 \end{aligned}$$

The interest, \$12.50, earned in the first quarter is added to \$1000. The principal becomes \$1012.50.

$$\begin{aligned} \text{Second quarter: } I &= 1012.50 \times 0.05 \times \frac{1}{4} \\ I &= 12.65625 \quad \text{The interest in the second} \\ &\quad \text{quarter is } \$12.66. \end{aligned}$$

The balance after two quarters is $\$1012.50 + 12.66$ or $\$1025.16$.

Answer each of the following questions.

1. How much interest is earned in the third quarter of Ms. Tanner's account?
2. What is the balance in her account after three quarters?
3. What is the balance in her account after one year?
4. Suppose Ms. Tanner's account is compounded semiannually. What is the balance at the end of six months?
5. What is the balance after one year if her account is compounded semiannually?

Study Guide**Probability and Odds**

You can measure the chances of an event happening with probability, a number that is always between 0 and 1, inclusive. You can express the probability of an event as a fraction, as a decimal, or as a percent, using the following relationship.

The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

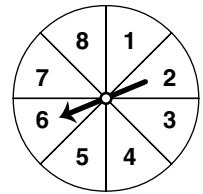
$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Example 1: The spinner at the right is spun. Find the probability of spinning a number greater than 5.

There are 3 favorable outcomes, spinning a 6, 7, or an 8. There are 8 possible outcomes.

$$\text{Therefore } P(\text{a number greater than 5}) = \frac{3}{8}.$$

Another way to measure the chance of an event is with odds.



The odds of an event is a ratio that compares the number of favorable outcomes to the number of unfavorable outcomes.

$$\text{Odds} = \frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$$

Example 2: Refer to the spinner above. Find the odds of spinning an even number.

There are 4 favorable outcomes: 2, 4, 6, or 8. The number of unfavorable outcomes is $8 - 4 = 4$. Therefore the odds of spinning an even number are 4:4 or 1:1.

Refer to the spinner above. Find the probability of each outcome.

- | | |
|-------------------------|----------------------------|
| 1. a 3 | 2. an even number |
| 3. a number less than 3 | 4. a number greater than 4 |

Refer to the spinner above. Find the odds of each outcome.

- | | |
|------------------|----------------------------|
| 5. an odd number | 6. a multiple of 3 |
| 7. not a 6 | 8. a number greater than 4 |

Practice**Probability and Odds**

Find the probability of each outcome if a pair of dice are rolled. Refer to the table below, which shows all of the possible outcomes when you roll a pair of dice.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

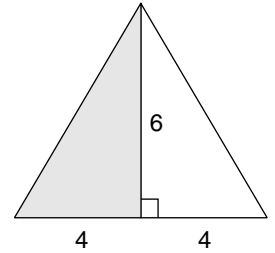
1. an even number on the second die
2. a sum of 8
3. a sum of 7
4. an odd sum
5. a sum less than 6
6. a sum greater than 7
7. both die are the same number
8. a sum less than 2

Find the odds of each outcome if a bag contains 7 blue marbles, 3 yellow marbles, and 2 red marbles.

9. choosing a blue marble
10. choosing a red marble
11. choosing a yellow marble
12. choosing a yellow or red marble
13. choosing a yellow or blue marble
14. choosing a blue or red marble
15. not choosing a blue or red marble
16. not choosing a blue marble

Geometric Probability

If a dart, thrown at random, hits a triangular board shown at the right, what is the probability that it will hit the shaded region? This probability can be determined by comparing the area of the shaded region to the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.



$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)}$$

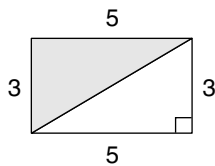
$$= \frac{12}{24} \text{ or } \frac{1}{2}$$

In general, if S is a subregion of some region R , then the probability, $P(S)$, that a point, chosen at random, belongs to subregion S is given by the following.

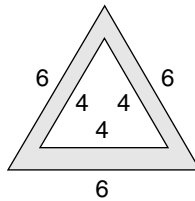
$$P(S) = \frac{\text{area of subregion } S}{\text{area of region } R}$$

Find the probability that a point, chosen at random, belongs to the shaded subregions of the following regions.

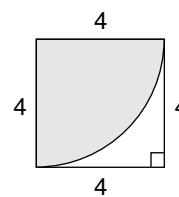
1.



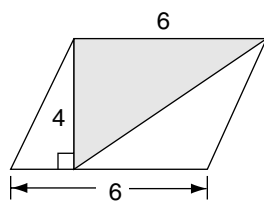
2.



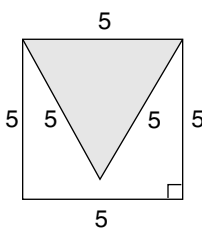
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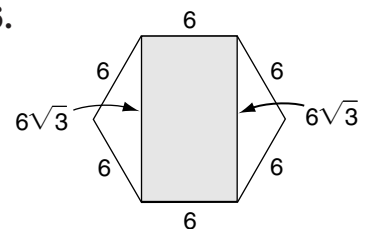
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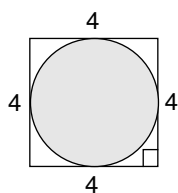
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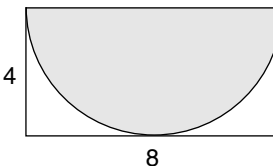
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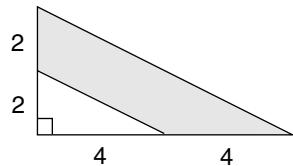
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8.



9.



Study Guide**Compound Events**

To win a game, you have to roll either an even number or a 3 with one die. What is the probability that you will win? The probability of tossing an even number = $P(\text{even number}) = \frac{1}{2}$, and the probability of tossing a 3 is $P(3) = \frac{1}{6}$. These events cannot occur at the same time and so they are **mutually exclusive**. This is a compound event because there is more than 1 event occurring. You find the probability of mutually exclusive events by adding the probabilities of each event.

$$\begin{aligned} P(\text{even number or a 3}) &= P(\text{even number}) + P(3) \\ &= \frac{1}{2} + \frac{1}{6} \text{ or } \frac{2}{3} \end{aligned}$$

Compound events are called **independent** if the outcome of one event does not affect the outcome of the other event. For example, if two dice are rolled, rolling an odd number with the first die does not affect rolling a 4 on the second die. You find the probability of two independent events by multiplying the probabilities of each event.

$$\begin{aligned} P(\text{odd number and a 4}) &= P(\text{odd number}) \cdot P(4) \\ &= \frac{1}{2} \cdot \frac{1}{6} \text{ or } \frac{1}{12} \end{aligned}$$

Probability of Mutually Exclusive Events A and B	$P(A \text{ or } B) = P(A) + P(B)$
Probability of Independent Events A and B	$P(A \text{ and } B) = P(A) \cdot P(B)$

A die is rolled. Find the probability of each compound event.

- $P(\text{a 4 or a 2})$
- $P(\text{an odd number or a 1})$
- $P(\text{a 2 or a 3})$
- $P(\text{a 1 or a 6})$
- $P(\text{an even number or a 5})$
- $P(\text{a 1 or a multiple of 2})$
- $P(\text{a 1 or a number greater than 4})$
- $P(\text{an odd number or an even number})$

Two dice are rolled. Find the probability of each compound event.

- $P(\text{an odd number and a 1})$
- $P(\text{an odd number and an even number})$
- $P(\text{two odd numbers})$
- $P(\text{a 6 and a number greater than 4})$
- $P(\text{two numbers less than 3})$
- $P(\text{an even number and a 5})$

Compound Events

Two dice are rolled. Find the probability of each outcome.

1. $P(\text{even number and } 2)$
2. $P(5 \text{ and } 5)$
3. $P(\text{odd number and a number less than } 6)$
4. $P(3 \text{ and a number less than } 3)$
5. $P(\text{even number and a number greater than } 2)$
6. $P(6 \text{ and a number greater than } 2)$

A card is drawn from a standard deck of cards. Determine whether the evens are mutually exclusive or inclusive. Then find each probability.

7. $P(\text{jack or five})$
8. $P(\text{ace or club})$
9. $P(\text{red card or four})$
10. $P(\text{face card or black card})$
11. $P(\text{spade or diamond})$
12. $P(\text{black card or odd-numbered card})$
13. $P(\text{heart or black card})$
14. $P(\text{heart or even-numbered card})$
15. $P(\text{face card or diamond})$
16. $P(\text{red card or black card})$
17. $P(\text{even-numbered card or ace})$
18. $P(\text{red card or heart})$

Conditional Probability

The probability of an event given the occurrence of another event is called **conditional probability**. The conditional probability of event A given event B is denoted $P(A|B)$.

Example: Suppose a pair of number cubes is rolled. It is known that the sum is greater than seven. Find the probability that the number cubes match.

There are 15 sums greater than seven and there are 36 possible pairs altogether.

$$P(B) = \frac{15}{36}$$

There are three matching pairs greater than seven, (4, 4), (5, 5), and (6, 6).

$$P(A \text{ and } B) = \frac{3}{36}$$

$$\begin{aligned} P(A|B) &= \frac{P(A \text{ and } B)}{P(B)} \\ &= \frac{\frac{3}{36}}{\frac{15}{36}} \text{ or } \frac{1}{5} \end{aligned}$$

The conditional probability is $\frac{1}{5}$.

A card is drawn from a standard deck of 52 cards and is found to be red. Given that event, find each of the following probabilities.

1. $P(\text{heart})$
2. $P(\text{ace})$
3. $P(\text{face card})$
4. $P(\text{jack or ten})$
5. $P(\text{six of spades})$
6. $P(\text{six of hearts})$

A sports survey taken at Stirers High School shows that 48% of the respondents liked soccer, 66% liked basketball, and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey and 28% liked soccer and hockey. Finally, 12% liked all three sports.

7. Find the probability that Meg likes soccer if she likes basketball.
8. Find the probability that Juan likes basketball if he likes soccer.
9. Find the probability that Mieko likes hockey if she likes basketball.
10. Find the probability that Greg likes hockey if he likes soccer.

Scaling (Technical Artist)

Have you ever wondered how people produce those fancy charts, graphs, and technical diagrams in books and magazines? They do not mysteriously appear. Rather, technical artists equipped with computer drawing software produce them at the keyboard. Among the geometric transformations they use is the dilation, or scaling operation. Critical to scaling a figure is the choice of what scale factor to use.

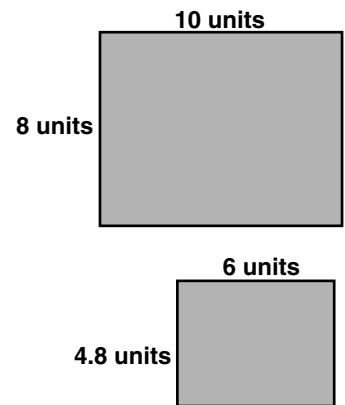
The rectangle at the right is 10 units long and 8 units wide. If a technical artist wants to make a scaled copy using a scale factor of 60%, what will be the dimensions of the scaled copy?

Find 60% of each dimension.

$$\text{Length: } 60\% \text{ of } 10 = 0.6 \times 10 \text{ or } 6 \text{ units}$$

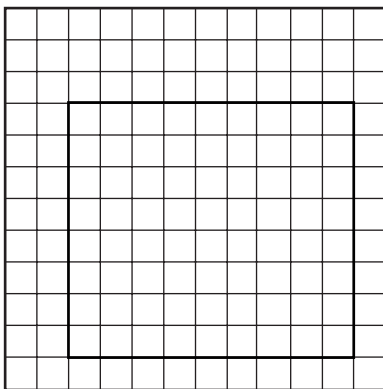
$$\text{Width: } 60\% \text{ of } 8 = 0.6 \times 8 \text{ or } 4.8 \text{ units}$$

The scaled copy of the given rectangle is shown at the right. It will be 6 units long and 4.8 units wide.

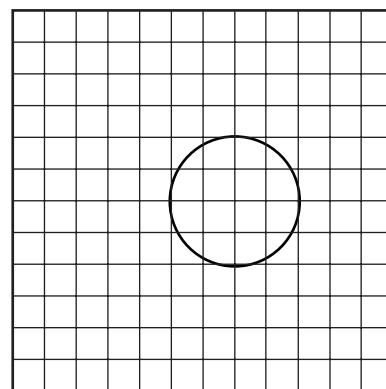


On the grid given, sketch the required enlargement or reduction of the figure shown.

1. Scale: 50%



2. Scale: 150%



3. Do you get a larger figure or a smaller figure if the scale is more than 100%? less than 100%