



Study Guide

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Relations

A set of ordered pairs is called a **relation**. The set of all first coordinates of the ordered pairs is the **domain** of the relation. The set of all second coordinates is the range. You can use a table or a graph to represent a relation.

Example 1: Express the relation $\{(-3, 1), (0, 3), (1, -1), (3, -2)\}$ as a table and as a graph. Then determine the domain and range.

The domain is $\{-3, 0, 1, 3\}$ and the range is $\{1, 3, -1, -2\}$.

X	У
-3	1
0	3
1	-1
3	-2



Example 2: Express the relation shown on the graph as a set of ordered pairs. Then find the domain and the range.

The set of ordered pairs for the relation is $\{(-2, 0), (-1, 3), (2, -1), (3, -3), (4, 1)\}.$ The domain is $\{-2, -1, 2, 3, 4\}$ and the range is $\{0, 3, -1, -3, 1\}$.

Identify the domain and the range of each function.

1. $\{(-6, 0), (-2, 3), (4, -1)\}$

x	y
-4	-3
-3	-3
-2	1
1	4
2	6

2.

3. Express the relation

 $\{(-4, -1), (-2, -2), (0, 0)\}$ as a table and as a graph. Then determine the domain and the range.

				V	
-					
-			0		X
			•	r	

4. Express the relation shown on the graph as as a set of ordered pairs and in a table. Then determine the domain and the range.







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Relations

Express each relation as a table and as a graph. Then determine the domain and the range.





(4.5, -1)



3. $\{(-5, 3.5), (-3, -4), (1.5, -5), (3, 3),$ **4.** $\{(-3.9, -2), (0, 4.5), (2.5, -5), (4, 0.5)\}$



Express each relation as a set of ordered pairs and in a table. Then determine the domain and the range.







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The Tower of Hanoi

The Tower of Hanoi puzzle has three pegs, with a stack of disks on peg a. The object is to move all of the disks to another peg. You may move only one disk at a time. Also, a larger disk may never be put on top of a smaller disk.

A chart has been started to record your moves as you solve the puzzle.

Another way to record the moves is to use letters. For example, the first two steps in the chart can be recorded as 1c, 2b. This shows that disk 1 is moved to peg c, and then disk 2 is moved to peg b.

Solve each problem.

- 1. Finish the chart to solve the Tower of Hanoi puzzle for three disks.
- 2. Record your solution using letters.
- **3.** On a separate sheet of paper, solve the puzzle for four disks. Record your solution.
- 4. Solve the puzzle for five disks. Record your solution.
- **5.** If you start with an odd number of disks and you want to end with the stack on peg c, what should be your first move?
- 6. If you start with an even number of disks and you want to end with the stack on peg b, what should be your first move?



Peg b	Peg c
	1
2	1
	Peg b 2 2



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Equations as Relations

Since x + 5 = 9 is true when x = 4, we say that 4 is a solution of the equation. Equations with two variables have solutions that may include one or more ordered pairs. The ordered pair (-3, 0) is a solution of the equation y = x + 3 because 0 = -3 + 3. But (-4, 1) is not solution because $1 \neq -4 + 3$.

Example: Solve y = -1 + x if the domain is $\{-3, -1, 0, 1\}$. Graph the solution.

Substitute each value of *x* into the equation to find the corresponding *y*-value. Then graph the ordered pairs.

X	У	(x, y)
-3	-1 + (-3) = -4	(-3, -4)
-1	-1 + (-1) = -2	(-1, -2)
0	-1 + 0 = -1	(0, -1)
1	-1 + 1 = 0	(1, 0)

					J			
						_		
*					(1	, u)	+
				0				X
_(-	-1	, –	2)		(0	, -	·1)	
_								
(–3	3, -	-4)						
					1			

The solution set is $\{(-3, -4), (-1, -2), (0, -1), (1, 0)\}$.

Which ordered pairs are solutions of each equation?

1. $y = x - 4$	a. (−1, −5)	b. (−1, −3)	c. (0, -4)	d. (5, 9)
2. $b = 2a + 9$	a. (7, 23)	b. (-2, 5)	c. (1, 11)	d. (−6, −3)
3. $4g + h = -6$	a. (-4, -10)	b. (-10, 34)	c. (0, 6)	d. (3, −18)
4. $-3x - y = 5$	a. (2, 11)	b. (2, −11)	c. (−4, −17)	d. (−10, −35)

Solve each equation if the domain is $\{-2, -1, 0, 1, 2\}$. Graph the solution set.









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Equations as Relations

Which ordered pairs are solutions of each equation?

1. $a + 3b = 5$	a. (2, 1)	b. (1, −2)	c. (-3, 3)	d. (8, −1)
2. $2g + 4h = 4$	a. (2, -2)	b. (4, −1)	c. (-2, 2)	d. (−4, 3)
3. $-3x + y = 1$	a. (4, 11)	b. (1, 4)	c. (-2, -5)	d. (−1, −2)
4. $9 = 5c - d$	a. (2, 1)	b. (1, −4)	c. (-2, -1)	d. (4, 11)
5. $2m = n + 6$	a. (4, -2)	b. (3, −2)	c. (3, 0)	d. (4, 2)

Solve each equation if the domain is $\{-2, -1, 0, 1, 2\}$. Graph the solution set.









Find the domain of each equation if the range is $\{-4, -2, 0, 1, 2\}$.

10. y = x + 5

11. 3y = 2x



Enrichment

Diophantine Equations

The first great algebraist, Diophantus of Alexandria (about A.D. 300), devoted much of his work to the solving of indeterminate equations. An indeterminate equation has more than one variable and an unlimited number of solutions. An example is x + 2y = 4.

When the coefficients of an indeterminate equation are integers and you are asked to find solutions that must be integers, the equation is called a **diophantine equation**. Such equations can be quite difficult to solve, often involving trial and error and some luck!

Solve each diophantine equation by finding at least one pair of positive integers that makes the equation true. Some hints are given to help you.

- 1. 2x + 5y = 32
 - **a.** First solve the equation for *x*.
 - **b.** Why must *y* be an even number?
 - c. Find at least one solution.
- **2.** 5x + 2y = 42
 - **a.** First solve the equation for *x*.
 - **b.** Rewrite your answer in the form x = 8 + some expression.
 - c. Why must (2 2y) be a multiple of 5?
 - **d.** Find at least one solution.

3.
$$2x + 7y = 29$$
 4. $7x + 5y = 118$

5.
$$8x - 13y = 100$$
 6. $3x + 4y = 22$

7.
$$5x - 14y = 11$$
 8. $7x + 3y = 40$

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Graphing Linear Relations

The solution set of the
equation $y = 2x - 1$
contains an infinite
number of ordered pairs.
A few of the solutions are
shown in the table at the
right.

x	У	(x, y)
-1	2(-1) - 1 = -3	(-1, -3)
0	2(0) - 1 = -1	(0, -1)
1	2(1) - 1 = 1	(1, 1)
2	2(2) - 1 = 3	(2, 3)



Graphing the ordered pairs indicates that the graph of the equation is a straight line. An equation whose graph is a straight line is a **linear equation**.

Example: Graph y = 3 - x.

Select at least three values for *x*. Determine the values for *y*.

Graph the ordered pairs and use them to draw a line.

X	У	(x, y)
-1	3 - (-1) = 4	(-1, 4)
0	3 - 0 = 3	(0, 3)
1	3 - 1 = 2	(1, 2)
2	3 - 2 = 1	(2, 1)



Graph each equation.

















x



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Graphing Linear Relations

Determine whether each equation is a linear equation. Explain. If an equation is linear, identify A, B, and C.

3. 4y - 2x = 2**1.** 2xy = 6**2.** 3x = y**5.** 4x + 5xy = 18**6.** x + 3y = 7**4.** x = -37. $\frac{2}{x} = 8$ **9.** $3x^2 + 4y = 2$ 8. 5y = x

Graph each equation.















15. 2x + 3y = 40 X

18. -4x + 4y = -8



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Enrichment

Taxicab Graphs

You have used a rectangular coordinate system to graph equations such as y = x - 1 on a coordinate plane. In a coordinate plane, the numbers in an ordered pair (x, y) can be any two real numbers.

A **taxicab plane** is different from the usual coordinate plane. The only points allowed are those that exist along the horizontal and vertical grid lines. You may think of the points as taxicabs that must stay on the streets.

The taxicab graph shows the equations y = -2 and y = x - 1. Notice that one of the graphs is no longer a straight line. It is now a collection of separate points.

Graph these equations on the taxicab plane at the right.

1. $y = x + 1$	2. $y = -2x + 3$
3. $y = 2.5$	4. $x = -4$

Use your graphs for these problems.

- 5. Which of the equations would have the same graph in both the usual coordinate plane and the taxicab plane?
- **6.** Describe the form of equations that have the same graph in both the usual coordinate plane and the taxicab plane.

In the taxicab plane, distances are not measured diagonally, but along the streets. Write the taxi-distance between each pair of points.

7. (0, 0) and (5, 2)	8. (0, 0) and (−3, 2)	9. (0, 0) and (2, 1.5)
10. (1, 2) and (4, 3)	11. $(2, 4)$ and $(-1, 3)$	12. (0, 4) and (-2, 0)

Draw these graphs on the taxicab grid at the right.

- 13. The set of points whose taxi-distance from (0, 0)is 2 units.
- 14. The set of points whose taxi-distance from (2, 1)is 3 units.









Notice	that $f(x)$	replac	ees y	in	this
exampl	e.				

We read f(x) as "*f* of *x*." If x = 3, then f(3) = 2 - 5(3) or -13.

If f(x) = 3x - 1, find f(-2). Example: f(x) = 3x - 1f(-2) = 3(-2) - 1 Replace x with -2. f(-2) = -7 Simplify.

Determine whether each relation is a function.

1. $\{(3, 1), (-2, 0), (-3, 5), (5, 6)\}$

3. $\{(-8, -4), (3, 2), (-8, -1), (7, 0)\}$

Equations that are functions can be written in **functional notation**.







4. $\{(9, 10), (-9, 10), (-4, 5), (-5, 4)\}$

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Equation	Functional Notation
y = 2 - 5x	f(x) = 2 - 5x

2. $\{(0, 1), (2, 1), (3, -1)\}$

A **function** is a special kind of relation in which each member of the domain is paired with exactly one member of the range. Study these examples.

Functions

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 $\{(-4, 7), (0, 1), (5, 1)\}$

This relation is a function because every first coordinate is matched with exactly one second coordinate.

X	У
7	0
-2	3
7	-8

This relation is *not* a

function because 7

is paired with two

y-values, 0 and -8.



This relation is a function because every *x*-value on the graph is paired with exactly one *y*-value.

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Practice

Functions

Determine whether each relation is a function.

1. $\{(-2, 1), (2, 0), (3, 6), (3, -4), (5, 3)\}$ **2.** $\{(-3, 2), (-2, 2), (1, 2), (-3, 1), (0, 3)\}$

3. $\{(-4, 1), (-2, 1), (1, 2), (3, 2), (0, 3)\}$

5. $\{(4, -1), (-1, 4), (1, 4), (3, -4), (-4, 3)\}$ **6.** $\{(-1, 0), (-2, 2), (1, -2), (3, 5), (1, 3)\}$

4. $\{(3, 3), (-2, -2), (5, 3), (1, -4), (2, 3)\}$





8.

X

2



Ο

У

-3

9.	x	у
	-4	3
	2	0
	1	4
	-3	5
	3	5



Use the vertical line test to determine whether each relation is a function.



0





If f(x) = 3x - 2, find each value.

16. <i>f</i> (4)	17. <i>f</i> (-2)	18. <i>f</i> (8)	19. <i>f</i> (−5)
20. <i>f</i> (1.5)	21. <i>f</i> (2.4)	22. $f(\frac{1}{3})$	23. $f\left(-\frac{2}{3}\right)$
24. <i>f</i> (<i>b</i>)	25. <i>f</i> (2 <i>g</i>)	26. <i>f</i> (-3 <i>c</i>)	27. <i>f</i> (2.5 <i>a</i>)

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Composite Functions

Three things are needed to have a function—a set called the *domain*, a set called the *range*, and a *rule* that matches each element in the domain with only one element in the range. Here is an example.



Suppose we have three sets A, B, and C and two functions described as shown below.

Rule: f(x) = 2x + 1Rule: g(y) = 3y - 4



$$g(y) = 3y - 4$$

 $g(3) = 3(3) - 4 = 5$

Let's find a rule that will match elements of set A with elements of set C without finding any elements in set B. In other words, let's find a rule for the **composite function** g[f(x)].

Since f(x) = 2x + 1, g[f(x)] = g(2x + 1). Since g(y) = 3y - 4, g(2x + 1) = 3(2x + 1) - 4, or 6x - 1. Therefore, g[f(x)] = 6x - 1.

Find a rule for the composite function g[f(x)].

1.
$$f(x) = 3x$$
 and $g(y) = 2y + 1$
2. $f(x) = x^2 + 1$ and $g(y) = 4y$

3.
$$f(x) = -2x$$
 and $g(y) = y^2 - 3y$
4. $f(x) = \frac{1}{x-3}$ and $g(y) = y^{-1}$

5. Is it always the case that g[f(x)] = f[g(x)]? Justify your answer.

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Direct Variation

A linear function that can be written in the form y = kx, where $k \neq 0$, is called a **direct variation**. In a direct variation, *y* varies directly as *x*.

Because a direct variation is a linear function whose solution contains (0, 0), the graph of a direct variation equation is a straight line that passes through the origin.

Example 1: Determine whether each function is a direct variation.

	a. $y = 10x$		b. $y = 2x + 1$	
	The function is a function. Since 0 is a solution of <i>y</i> Therefore, the fu direct variation a will pass through	a linear 0 = 10(0), (0, 0) = 10x. unction is a and the graph h the origin.	The function is a linear function. But since $0 \neq 2(0) +$ (0, 0) is not a solution of y = 2x + 1. Therefore, the function is not a direct variation and the graph will not pass through the origin.	
Example 2:	Assume that y vari Find x when $y = -$	es directly as <i>x</i> and 12.	y = 30 when $x = 5$.	
	Step 1 Find the co variation, k	nstant of 2.	Step 2 Use $k = 6$ t y = -12.	to find x when
	y = kx 30 = k(5)	When $y = 30$, $x = 5$.	y = kx $y = 6x$	Substitute 6 for k
	6 = k	Solve for k.	-12 = 6x	Substitute –12 for y.
	When $y = -12, x =$	-2.	x = -2	Solve for x.

Determine whether each equation is a direct variation.

1. $y = x$	2. $y = x + 2$	3. $y = 6$	4. $y = 4 - x$
5. $\frac{y}{x} = 8$	6. $y = -7x$	7. $x = -1$	8. $\frac{y}{2x} = 5$

Solve. Assume that y varies directly as x.

9. Find *x* when y = 24 if y = 18 when x = 6.

10. Find x when y = 6 if y = -8 when x = 4.



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Direct Variation

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Practice

Determine whether each equation is a direct variation. Verify the answer with a graph.

1. y = 3x

















Solve. Assume that y varies directly as x.

- **7.** If y = 14 when x = 5, find *x* when y = 28.
- **9.** If x = 9 when y = 18, find *x* when y = 24.
- 11. Find y when x = 3 if y = -3 when x = 6.

- 8. Find y when x = 5 if y = -6 when x = 2.
- **10.** If y = 36 when x = -6, find *x* when y = 54.
- **12.** Find *y* when x = 8 if y = 4 when x = 5.

Solve by using direct variation.

- **13.** If there are 4 quarts in a gallon, how many quarts are in 4.5 gallons?
- 14. How many feet are in 62.4 inches if there are 12 inches in a foot?
- 15. If there are 2 cups in a pint, how many cups are in 7.2 pints?



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nth Power Variation

An equation of the form $y = kx^n$, where $k \neq 0$, describes an *n*th power variation. The variable *n* can be replaced by 2 to indicate the second power of *x* (the square of *x*) or by 3 to indicate the third power of *x* (the cube of *x*).

Assume that the weight of a person of average build varies directly as the cube of that person's height. The equation of variation has the form $w = kh^3$.

The weight that a person's legs will support is proportional to the cross-sectional area of the leg bones. This area varies directly as the square of the person's height. The equation of variation has the form $s = kh^2$.

Answer each question.

- 1. For a person 6 feet tall who weighs 200 pounds, find a value for k in the equation $w = kh^3$.
- 3. Find the value for k in the equation $w = kh^3$ for a baby who is 20 inches long and weighs 6 pounds.
- **5.** For a person 6 feet tall who weighs 200 pounds, find a value for k in the equation $s = kh^2$.
- 7. According to the adult equation you found (Exercise 1), how much would an imaginary giant 20 feet tall weigh?

- **2.** Use your answer from Exercise 1 to predict the weight of a person who is 5 feet tall.
- 4. How does your answer to Exercise 3 demonstrate that a baby is significantly fatter in proportion to its height than an adult?
- 6. For a baby who is 20 inches long and weighs 6 pounds, find an "infant value" for k in the equation $s = kh^2$.
- 8. According to the adult equation for weight supported (Exercise 5), how much weight could a 20-foot tall giant's legs actually support?
- 9. What can you conclude from Exercises 7 and 8?

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Inverse Variation

Shauna wants to improve her running speed. As her speed increases, her time decreases.

rate · time = distance							
r t d r t d							
$\frac{1}{9}$ mile/minute	9 minutes	1 mile	$\frac{1}{7}$ mile/minute	7 minutes	1 mile		
$\frac{1}{8}$ mile/minute	8 minutes	1 mile	$\frac{1}{6}$ mile/minute	6 minutes	1 mile		

The equation rt = d is an example of an **inverse variation**. An inverse variation is described by an equation of the form xy = k, where $k \neq 0$. We say that y varies inversely as x.

Example: Suppose *y* varies inversely as *x* and y = 3 when x = 4. Find *y* when x = -12. **Step 1** Find the constant of **Step 2** Use k = 12 to find y when variation, k. x = -12.xv = kxy = k3(4) = k When y = 3, xy = 12 Substitute 12 x = 4.for k. 12 = k Solve for k. -12y = 12 Substitute -12for x. y = -1 Solve for y. When x = -12, y = -1.

Determine if each equation is an inverse variation or a direct variation.

2. ab = 53. xy = -1**1.** y = 5x

Solve. Assume that y varies inversely as x.

- 4. Find y when x = -9 if y = 3 when x = 6.
- 5. Suppose y = 5 when x = 16. Find x when y = 10.

6. If y = 4.5 when x = 6, find y when x = 3.

- 7. Find y when x = 0.125 if y = 1.5 when x = 2.5.
- 8. Find x when y = 0.9, if y = 1.5 when x = 0.3.
- **9.** Suppose x = -8 when y = 6. Find y when x = 16.

10. If
$$y = \frac{1}{4}$$
 when $x = 8$, find x when $y = \frac{2}{5}$.



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Inverse Variation

Solve. Assume that y varies inversely as x.

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- **1.** Suppose y = 9 when x = 4. Find y when x = 12.
- **2.** Find x when y = 4 if y = -4 when x = 6.
- 3. Find x when y = 7 if y = -2 when x = -14.
- **4.** Suppose y = -2 when x = 8. Find y when x = 4.
- **5.** Suppose y = -9 when x = 2. Find y when x = -3.
- 6. Suppose y = 22 when x = 3. Find y when x = -6.
- 7. Find x when y = 9 if y = -3 when x = -18.
- 8. Suppose y = 5 when x = 8. Find y when x = 4.
- **9.** Find x when y = 15 if y = -6 when x = 2.5.
- **10.** If y = 3.5 when x = 2, find y when x = 5.
- **11.** If y = 2.4 when x = 5, find *y* when x = 6.
- **12.** Find *x* when y = -10 if y = -8 when x = 12.
- **13.** Suppose y = -3 when x = -0.4. Find y when x = -6.
- **14.** If y = -3.8 when x = -4, find *y* when x = 2.

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Analyzing Data

Fill in each table below. Then write inversely, or directly to complete each conclusion.

1.	1	2	4	8	16	32
	w	4	4	4	4	4
	Α					

For a set of rectangles with a width

of 4, the area varies _____ as the length.

3.	Oat bran	$\frac{1}{3}$ cup	$\frac{2}{3}$ cup	1 cup
	Water	1 cup	2 cup	3 cup
	Servings	1	2	

The number of servings of oat bran varies ______ as the number of cups of oat bran.

5.	Miles	100	100	100	100
	Rate	20	25	50	100
	Hours	5			

For a 100-mile car trip, the time the trip takes varies ______ as the average rate of speed the car travels.

Use the table at the right.

- **7.** *x* varies _____ as *y*.
- 8. *z* varies ______ as *y*.
- **9.** *x* varies _____ as *z*.

2.	Hours	2	4	5	6
	Speed	55	55	55	55
	Distance				

For a car traveling at 55 mi/h, the

distance covered varies _____ as the hours driven.

1.	Hours of Work	128	128	128
	People Working	2	4	8
	Hours per Person			

A job requires 128 hours of work. The number of hours each person works

varies ______ as the number of people working.

6.	b	3	4	5	6
	h	10	10	10	10
	Α	15			

For a set of right triangles with a height of 10, the area varies

_____ as the base.

x	1	1.5	2	2.5	3
У	2	3	4	5	6
z	60	40	30	24	20



School-to-Workplace

Traffic Patterns (Traffic Technician)

Highway planners often find a need to expand an existing highway system. Another bridge over a bay may need to be built. An existing roadway may need to be widened. What was once a country road may need to become a super highway.

One way to estimate how many cars and trucks use a particular route is to count them. Traffic technicians can actually count them and record results on paper. They can also use automatic devices to achieve the same goal. In any case, a statistical record of traffic patterns is necessary to determine whether a highway system needs to be expanded.

The table at the right shows the actual count of cars and trucks that use a particular roadway during rush hour on one weekday selected at random.

What relations does the table suggest? What are the domains and ranges of the relations?

There is a relationship between time intervals and the count of cars.

Domain: set of half-hour time intervals between 6:00 A.M. and 9:00 A.M. Range: {133, 200, 210, 222, 250}

There is a relationship between time intervals and the count of trucks.

Domain: set of half-hour time intervals between 6:00 A.M. and 9:00 A.M. Range: {50, 55, 60, 63, 65}

Solve.

- 1. In what time interval is the count of cars the greatest?
- 2. Is there a relationship between the number of cars using the road at one time and the number of trucks using the road at the same time?
- **3.** Is there a trend in the count of cars as rush hour progresses?

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Time	Cars	Trucks
6:00 a.m6:30 a.m.	133	65
6:30 a.m7:00 a.m.	200	60
7:00 а.м7:30 а.м.	210	63
7:30 а.м8:00 а.м.	210	63
8:00 a.m8:30 a.m.	250	55
8:30 a.m9:00 a.m.	222	50

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