

Study Guide

Slope

Slope is the ratio of the rise, or the vertical change, to the run, or the horizontal change. A greater ratio indicates a steeper slope.

A typical ski mountain has a slope of about $\frac{1}{4}$, while a car windshield may have a slope of 3.

For any two points (x_1, y_1) and (x_2, y_2) ,

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples: Find the slope of the line containing each pair of points.

a. $(4, -2)$ and $(-3, 7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - (-2)}{-3 - 4}$$

$$m = \frac{-9}{7}$$

b. $(-3, 6)$ and $(-3, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - 6}{-3 - (-3)}$$

$$m = \frac{-7}{0}$$

Since you cannot divide by 0, the slope is *undefined*.

c. $(4, 5)$ and $(-2, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 5}{-2 - 4}$$

$$m = \frac{0}{-6} \text{ or } 0$$

Determine the slope of the line passing through the points whose coordinates are listed.

1. $(-2, 1)$ and $(4, 2)$

2. $(0, 3)$ and $(4, 1)$

3. $(-3, -5)$, $(5, 7)$

4. $(4, 3)$ and $(4, -1)$

5. $(8, -2)$ and $(-3, -2)$

6. $(5, 1)$ and $(-1, -5)$

7. $(7, -1)$ and $(6, 6)$

8. $(5, -2)$ and $(-5, 2)$

9. $(7, -7)$ and $(-6, 6)$

10. $(4, -4)$ and $(0, 3)$

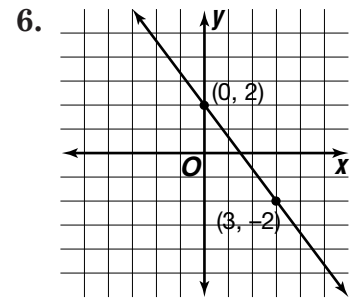
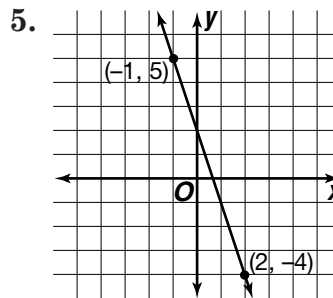
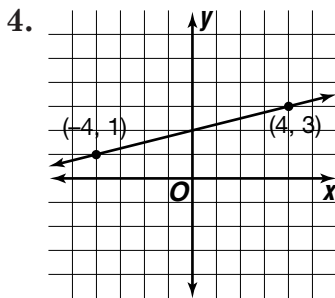
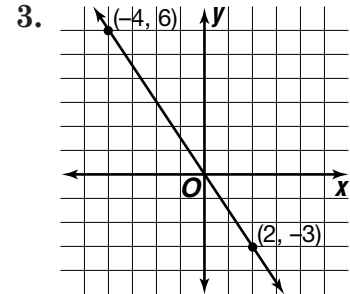
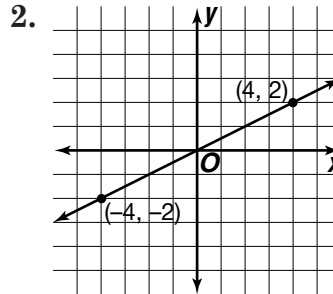
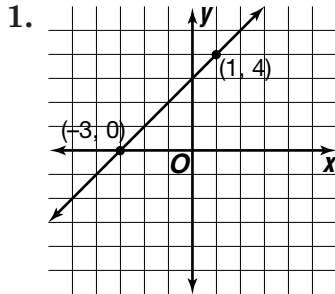
11. $(-2, 4)$ and $(-2, 9)$

12. $(0, 8)$ and $(-3, 8)$

Practice

Slope

Determine the slope of each line.



Determine the slope of the line passing through the points whose coordinates are listed in each table.

7.

x	y
-1	-3
0	0
1	3
2	6

8.

x	y
-2	5
2	4
6	3
10	2

9.

x	y
-3	4
-1	5
1	6
3	7

Determine the slope of each line.

10. the line through points at (3, 4) and (4, 6)

11. the line through points at (-3, -2) and (-2, -5)

12. the line through points at (2, 3) and (-5, 1)

13. the line through points at (4, -1) and (9, 6)

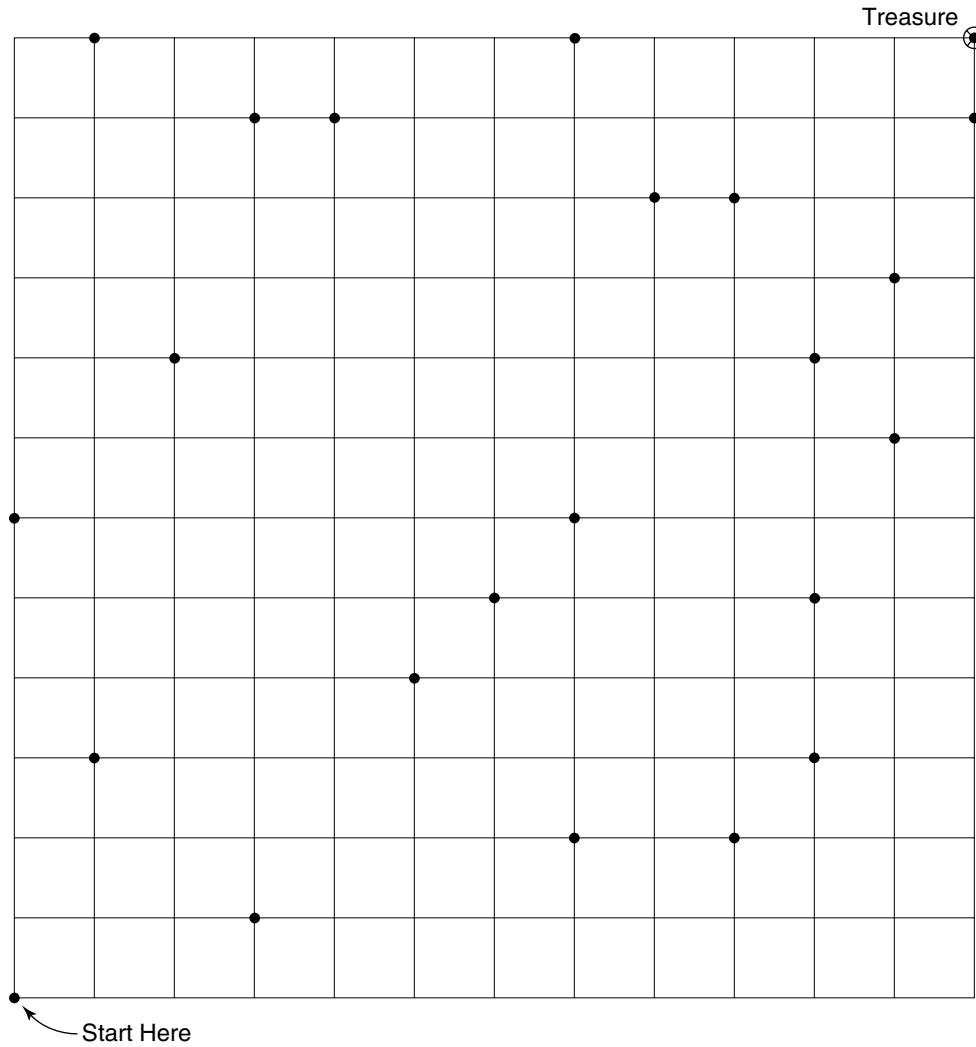
14. the line through points at (-4, 4) and (-9, -8)

15. the line through points at (-6, 2) and (7, -3)

Enrichment

Treasure Hunt with Slopes

Using the definition of slope, draw lines with the slopes listed below. A correct solution will trace the route to the treasure.



- | | | | | | |
|-------------|------------------|-------------------|-------------------|--------------------|-------|
| 1. 3 | 2. $\frac{1}{4}$ | 3. $-\frac{2}{5}$ | 4. 0 | 5. 1 | 6. -1 |
| 7. no slope | 8. $\frac{2}{7}$ | 9. $\frac{3}{2}$ | 10. $\frac{1}{3}$ | 11. $-\frac{3}{4}$ | 12. 3 |

Study Guide

Writing Equations in Point-Slope Form

You can write the equation of a line if you know its slope and the coordinates of one point or if you know the coordinates of two points on the line. Use the point-slope form.

Point-Slope Form

For a nonvertical line through the point at (x_1, y_1) with slope m , the point-slope form of a linear equation is

$$y - y_1 = m(x - x_1).$$

Examples: Write the point-slope form of an equation for each line.

- a. the line passing through the point at $(-4, 2)$ and having a slope of $\frac{2}{3}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{2}{3}(x - (-4))$$

$$y - 2 = \frac{2}{3}(x + 4)$$

- b. the line passing through points at $(4, -4)$ and $(-3, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Find } m.$$

$$m = \frac{1 - (-4)}{-3 - 4} \text{ or } -\frac{5}{7}$$

$$y - (-4) = -\frac{5}{7}(x - 4) \quad \text{Substitute } m.$$

$$y + 4 = -\frac{5}{7}(x - 4)$$

Write the point-slope form of an equation for each line, given either the coordinates of a point and the slope or the coordinates of two points.

1. $(-2, -1), m = 2$

2. $(4, -1), m = -\frac{1}{2}$

3. $(-3, -5), m = \frac{3}{2}$

4. $(4, 3), m = \frac{1}{5}$

5. $(8, -2), m = 0$

6. $(5, 1), m = -\frac{2}{3}$

7. $(7, -1)$ and $(6, 6)$

8. $(5, -2)$ and $(-5, 2)$

9. $(7, -7)$ and $(-6, 6)$

10. $(4, -4)$ and $(0, 3)$

11. $(-2, -4)$ and $(-12, 9)$

12. $(0, 8)$ and $(-3, 8)$

Practice

Writing Equations in Point-Slope Form

Write the point-slope form of an equation for each line passing through the given point and having the given slope.

1. $(4, 7), m = 3$

2. $(-2, 3), m = 5$

3. $(6, -1), m = -2$

4. $(-5, -2), m = 0$

5. $(-4, -6), m = \frac{2}{3}$

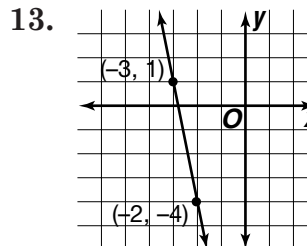
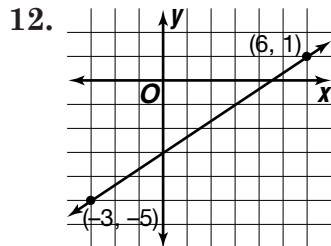
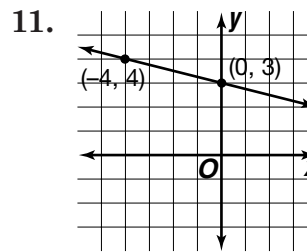
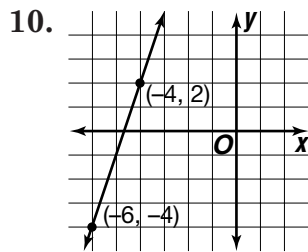
6. $(-8, 3), m = -\frac{3}{5}$

7. $(7, -9), m = 4$

8. $(-6, 3), m = -\frac{1}{2}$

9. $(-2, -5), m = 8$

Write the point-slope form of an equation for each line.



14. the line through points
at $(-2, -2)$ and $(-1, -6)$

15. the line through points
at $(-7, -3)$ and $(5, -1)$

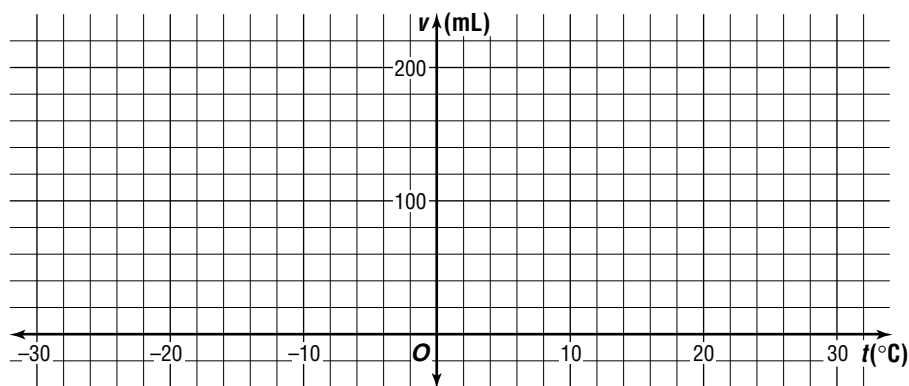
Enrichment***Celsius and Kelvin Temperatures***

If you blow up a balloon and put it in the refrigerator, the balloon will shrink as the temperature of the air in the balloon decreases.

The volume of a certain gas is measured at 30° Celsius. The temperature is decreased and the volume is measured again.

Temperature (t)	Volume (v)
30°C	202 mL
21°C	196 mL
0°C	182 mL
-12°C	174 mL
-27°C	164 mL

- Graph this table on the coordinate plane provided below.



- Find the equation of the line that passes through the points you graphed in Exercise 1.
- Use the equation you found in Exercise 2 to find the temperature that would give a volume of zero. This temperature is the lowest one possible and is called “absolute zero.”
- In 1848 Lord Kelvin proposed a new temperature scale with 0 being assigned to absolute zero. The size of the degree chosen was the same size as the Celsius degree. Change each of the Celsius temperatures in the table above to degrees Kelvin.

Study Guide

Writing Equations in Slope-Intercept Form

Given the slope m and the y -intercept b of a line, the slope-intercept form of an equation of the line is $y = mx + b$. Sometimes it is more convenient to express the equation of a line in slope-intercept form instead of point-slope form. This form is especially useful when graphing lines.

Examples: Write the point-intercept form of an equation for each line.

- a. the line with $m = -\frac{2}{3}$
and $b = 4$

$$y = mx + b$$

$$y = -\frac{2}{3}x + 4$$

- b. the line passing through points
at $(4, -2)$ and $(6, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - (-2)}{6 - 4} = \frac{4}{2} = 2$$

$$y - 2 = 2(x - 6)$$

$$y - 2 = 2x - 12$$

$$y - 2 + 2 = 2x - 12 + 2$$

$$y = 2x - 10$$

Find m .

Multiply.

Add 2 to each side.

Simplify.

Write the slope-intercept form of an equation for each line, given either the slope and the y -intercept or the coordinates of two points.

1. $m = 2, b = -3$

2. $m = \frac{1}{2}, b = 5$

3. $m = \frac{7}{4}, (0, -3)$

4. $m = -\frac{1}{5}, b = 0$

5. $m = \frac{1}{6}, b = -1$

6. $m = 0, b = 5$

7. $(5, -3)$ and $(-4, -3)$

8. $(-5, 1)$ and $(-1, 5)$

9. $(0, -1)$ and $(6, 5)$

10. $(-2, -2)$ and $(-4, 2)$

11. $(-2, -4)$ and $(-12, 6)$

12. $(0, 8)$ and $(-3, 4)$

13. $(0, 8)$ and $(-2, 7)$

14. $(0, -3)$ and $(6, 5)$

Practice**Writing Equations in Point-Intercept Form**

Write an equation in slope-intercept form of the line with each slope and y-intercept.

1. $m = -3, b = 5$

2. $m = 6, b = 2$

3. $m = 4, b = -1$

4. $m = 0, b = 4$

5. $m = \frac{2}{5}, b = -7$

6. $m = -\frac{3}{4}, b = 8$

7. $m = -\frac{4}{3}, b = -2$

8. $m = -5, b = 6$

9. $m = \frac{1}{2}, b = -9$

Write an equation in slope-intercept form of the line having the given slope and passing through the given point.

10. $m = 3, (4, 2)$

11. $m = -2, (-1, 3)$

12. $m = 4, (0, -7)$

13. $m = -\frac{3}{5}, (-5, -3)$

14. $m = \frac{1}{4}, (-8, 6)$

15. $m = -\frac{2}{3}, (9, -4)$

16. $m = \frac{5}{6}, (6, -6)$

17. $m = 0, (-8, -7)$

18. $m = -\frac{3}{2}, (-8, 9)$

Write an equation in slope-intercept form of the line passing through each pair of points.

19. $(1, 3)$ and $(-3, -5)$

20. $(0, 5)$ and $(3, -4)$

21. $(2, 1)$ and $(3, 6)$

22. $(-3, 0)$ and $(6, -6)$

23. $(4, 5)$ and $(-5, 5)$

24. $(0, 6)$ and $(-4, 3)$

25. $(-3, 2)$ and $(3, -6)$

26. $(-7, -6)$ and $(-5, -3)$

27. $(6, -4)$ and $(0, 2)$

Enrichment***Ideal Weight***

You can find your ideal weight as follows.

A woman should weigh 100 pounds for the first 5 feet of height and 5 additional pounds for each inch over 5 feet (5 feet = 60 inches). A man should weigh 106 pounds for the first 5 feet of height and 6 additional pounds for each inch over 5 feet. These formulas apply to people with normal bone structures.

To determine your bone structure, wrap your thumb and index finger around the wrist of your other hand. If the thumb and finger just touch, you have normal bone structure. If they overlap, you are small-boned. If they don't overlap, you are large-boned. Small-boned people should decrease their calculated ideal weight by 10%. Large-boned people should increase the value by 10%.

Calculate the ideal weights of these people.

1. woman, 5 ft 4 in., normal-boned
2. man, 5 ft 11 in., large-boned
3. man, 6 ft 5 in., small-boned
4. you, if you are at least 5 ft tall

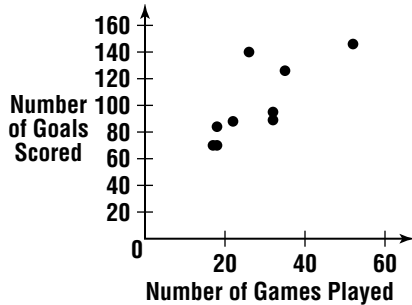
Suppose a normal-boned man is x inches tall. If he is at least 5 feet tall, then $x - 60$ represents the number of inches this man is over 5 feet tall. For each of these inches, his ideal weight is increased by 6 pounds. Thus, his proper weight (y) is given by the formula $y = 6(x - 60) + 106$ or $y = 6x - 254$. If the man is large-boned, the formula becomes $y = 6x - 254 + 0.10(6x - 254)$.

5. Write the formula for the weight of a large-boned man in slope-intercept form.
6. Derive the formula for the ideal weight (y) of a normal-boned female with height x inches. Write the formula in slope-intercept form.
7. Derive the formula in slope-intercept form for the ideal weight (y) of a large-boned female with height x inches.
8. Derive the formula in slope-intercept form for the ideal weight (y) of a small-boned male with height x inches.
9. Find the heights at which normal-boned males and large-boned females would weigh the same.

Study Guide

Scatter Plots

The scatter plot below is a graph of the number of games played in the highest-scoring World Cup finals compared to the number of goals scored. The data are listed in the table.

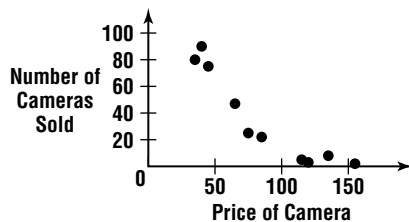


Year	Number of Games Played	Number of Goals Scored
1954	26	140
1938	18	84
1934	17	70
1950	22	88
1930	18	70
1958	35	126
1970	32	95
1982	52	146
1962	32	89
1966	32	89

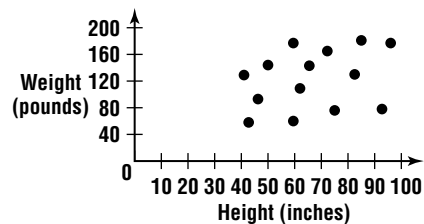
You can use the scatter plot to draw conclusions about the data. The data points fall roughly along a line with a positive slope. Therefore, we say there is a positive relationship between the number of games played and the number of goals scored. A line with a negative slope would indicate a negative relationship, and no line would indicate no relationship between the variables. A valid conclusion from this scatter plot is that as the number of games played increases, more goals are scored.

Determine whether each scatter plot has a positive relationship, negative relationship, or no relationship. If there is a relationship, describe it.

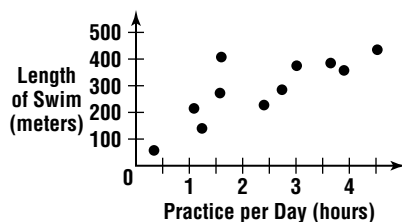
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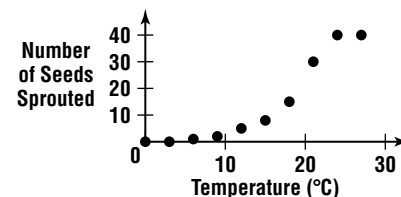
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3.



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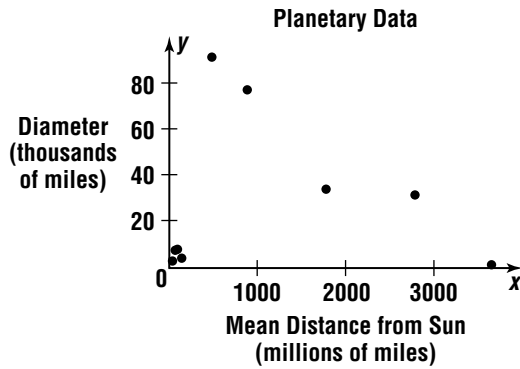


Practice

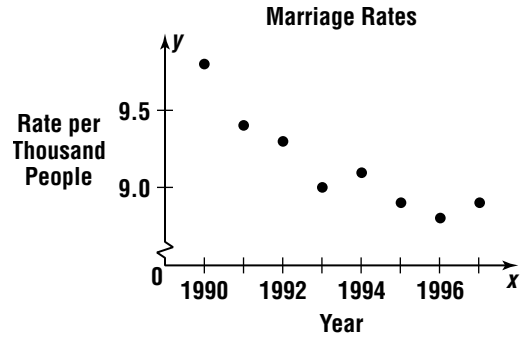
Scatter Plots

Determine whether each scatter plot has a positive relationship, negative relationship, or no relationship. If there is a relationship, describe it.

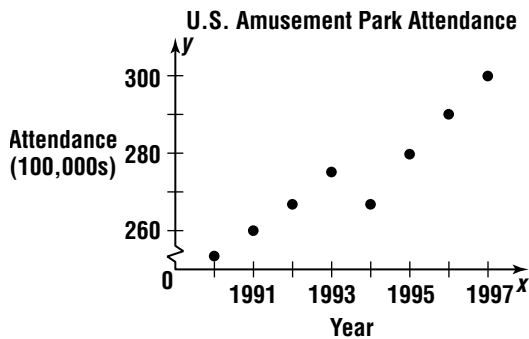
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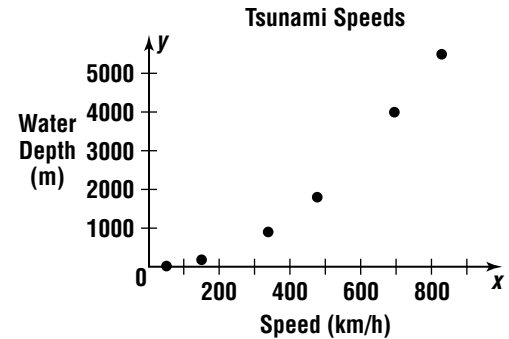
2.



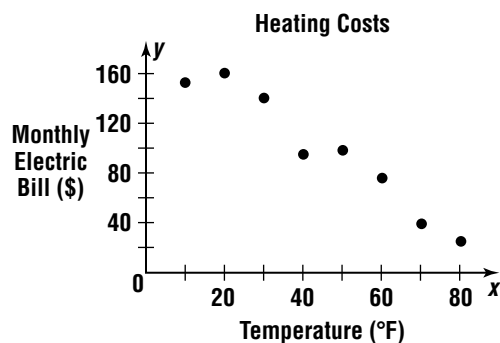
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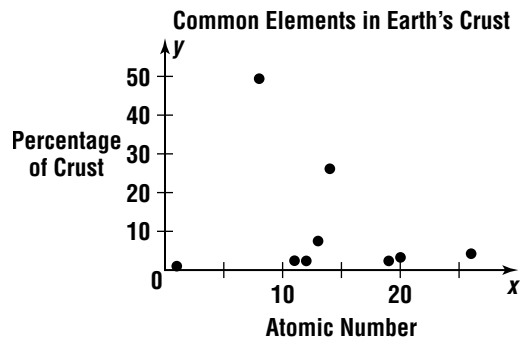
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5.



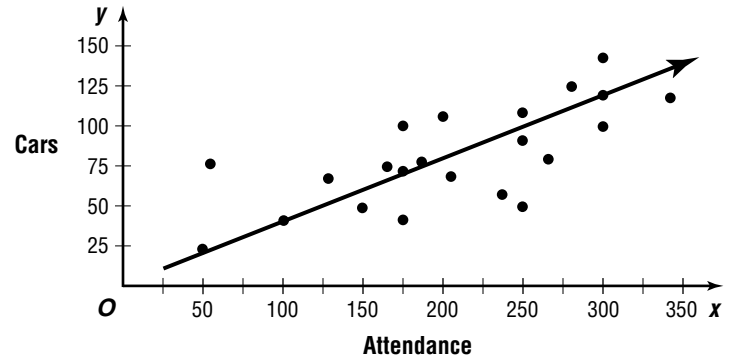
6.



Enrichment**A Scatter Plot**

Each point on the graph shows the relation between the number of people attending the Roxy Cinema and the number of cars in the parking lot.

A line is drawn that appears to lie close to most of the points. Here is how to find the equation of this line.



The line passes through (100, 40) and (300, 120). Use the slope-intercept form.

$$\begin{aligned} m &= \frac{120 - 40}{300 - 100} \\ &= \frac{80}{200} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} y &= mx + b \\ 40 &= \frac{2}{5}(100) + b \\ 0 &= b \end{aligned}$$

An equation for the line is $y = \frac{2}{5}x$.

Solve each problem.

- Suppose the owner of the Roxy decides to increase the seating capacity of the theater to 1000. How many cars should the parking lot be prepared to accommodate?
- The points (240, 60) and (340, 120) lie on the scatter plot. Write an equation for the line through these points.
- Do you think the equation in Exercise 2 is a good representation of the relationship in this problem?
- Suppose the equation for the relationship between attendance at the theater and cars in the parking lot is $y = 2x + 20$. What might you suspect about the users of the parking lot?

Study Guide

Graphing Linear Equations

You may use the slope-intercept form to graph linear equations, as shown in the example below.

Example: Graph $2x + 3y = 6$.

Rewrite in slope-intercept form.

$$2x + 3y = 6$$

$$2x + 3y - 2x = 6 - 2x \quad \text{Subtract } 2x \text{ from each side.}$$

$$3y = -2x + 6$$

$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3} \quad \text{Divide each side by 3.}$$

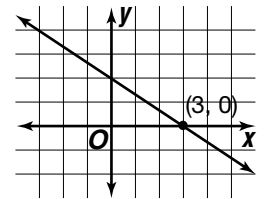
$$y = -\frac{2}{3}x + 2$$

To graph $y = -\frac{2}{3}x + 2$, plot a point at the y-intercept, 2.

Then use the slope, $-\frac{2}{3}$. From $(0, 2)$, go down 2 units.

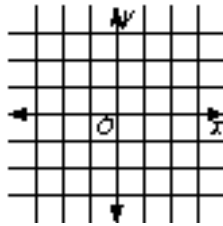
Then go right 3 units. Graph a point at $(3, 0)$.

Draw the line through the points.

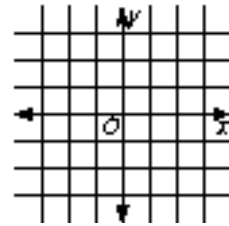


Graph each equation.

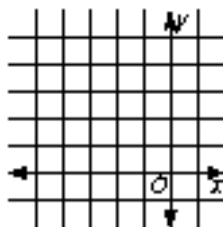
1. $y = -x - 1$



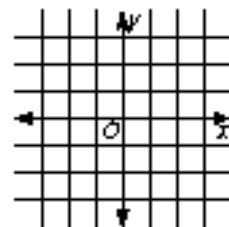
2. $y = \frac{4}{3}x - 2$



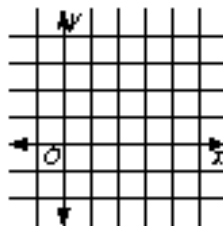
3. $y = \frac{1}{2}x + 4$



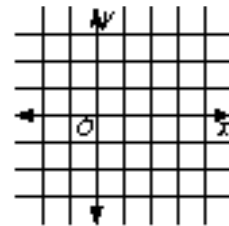
4. $y = -3$



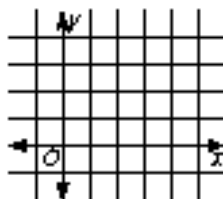
5. $2x + 4y = 8$



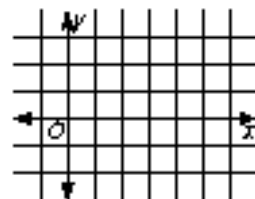
6. $2x - 3y = 6$



7. $x + 2y = 4$



8. $x - 5y = 5$

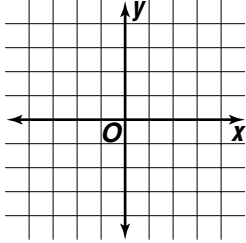


Practice

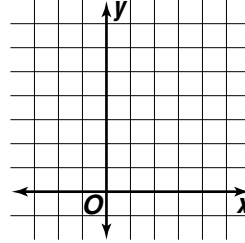
Graphing Linear Equations

Determine the x -intercept and y -intercept of the graph of each equation. Then graph the equation.

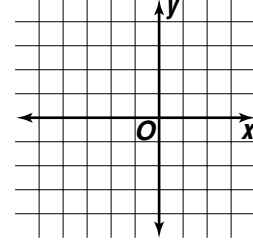
1. $x + y = -2$



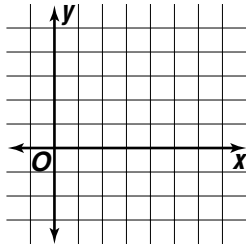
2. $2x + y = 6$



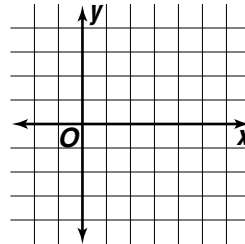
3. $x - 2y = -4$



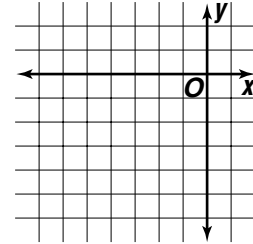
4. $2x + 3y = 12$



5. $3x - 3y = 9$

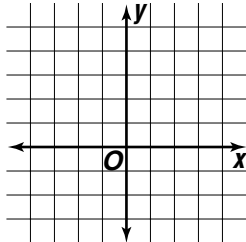


6. $5x + 6y = -30$

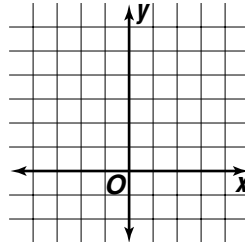


Determine the slope and y -intercept of the graph of each equation. Then graph the equation.

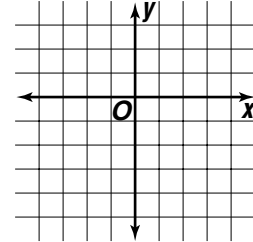
7. $y = -x + 3$



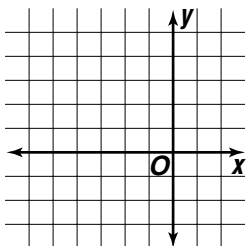
8. $y = 5$



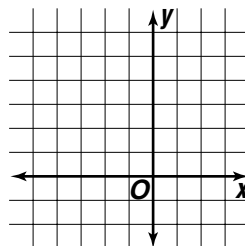
9. $y = 3x - 4$



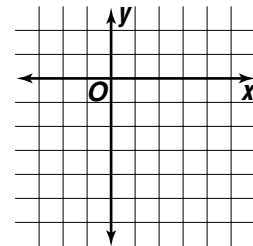
10. $y = \frac{2}{5}x + 2$



11. $y = -\frac{3}{4}x + 1$



12. $y = \frac{2}{3}x - 6$



Enrichment

Equations of Lines and Planes in Intercept Form

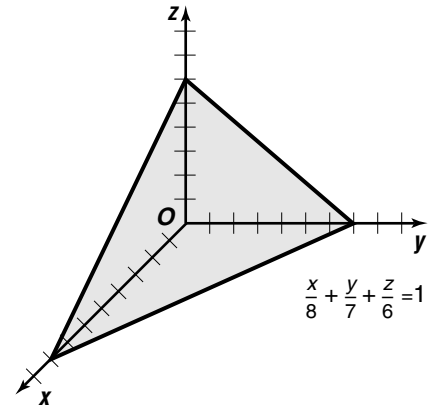
One form that a linear equation may take is **intercept form**. The constants a and b are the x - and y -intercepts of the graph.

$$\frac{x}{a} + \frac{y}{b} = 1$$

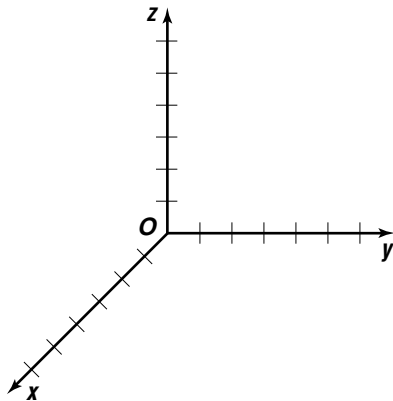
In three-dimensional space, the equation of a plane takes a similar form.

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

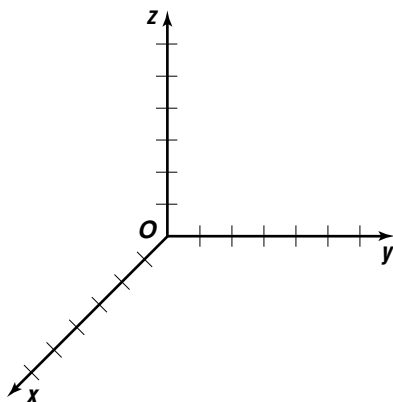
Here, the constants a , b , and c are the points where the plane meets the x , y , and z -axes.

**Solve each problem.**

1. Graph the equation $\frac{x}{3} + \frac{y}{2} + \frac{z}{1} = 1$.



5. Graph the equation $\frac{x}{1} + \frac{y}{4} + \frac{z}{2} = 1$.

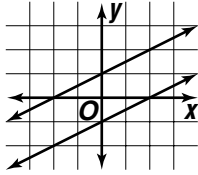


- For the plane in Exercise 1, write an equation for the line where the plane intersects the xy -plane. Use intercept forms.
- Write an equation for the line where the plane intersects the xz -plane.
- Write an equation for the line where the plane intersects the yz -plane.
- Write an equation for the xy -plane.
- Write an equation for the yz -plane.
- Write an equation for a plane parallel to the xy -plane with a z -intercept of 2.
- Write an equation for a plane parallel to the yz -plane with an x -intercept of -3 .

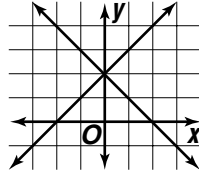
Study Guide

Families of Linear Graphs

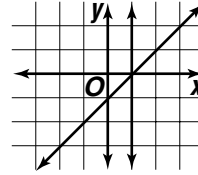
Graphs of linear equations that have at least one characteristic in common are called **families of graphs**. Graphs are families if they have the same slope, the same y -intercept, or the same x -intercept. An example of each is shown below.



same slope



same y -intercept



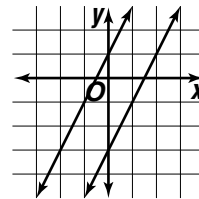
same x -intercept

Example: Graph the pair of equations. Explain why they are a family of graphs.

$$y = 2x - 3$$

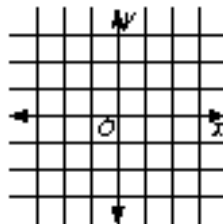
$$y = 2x + 1$$

Since both graphs have the same slope, this is a family of graphs.

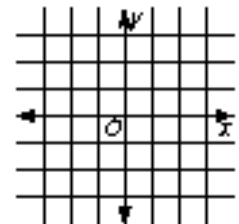


Graph each pair of equations. Explain why they are a family of graphs.

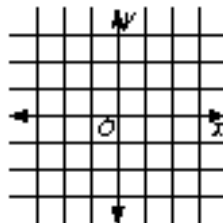
1. $y = -x - 1$
 $y = -x + 2$



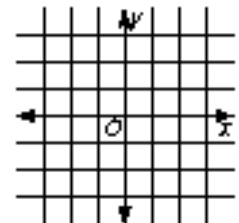
2. $y = \frac{4}{3}x - 2$
 $y = \frac{3}{4}x - 2$



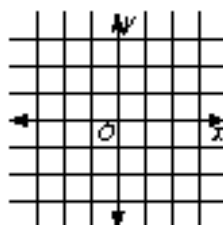
3. $y = x - 1$
 $y = -x + 1$



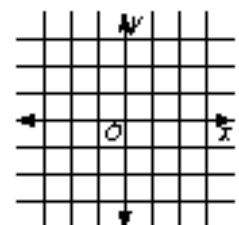
4. $2y = 3x - 6$
 $4y = 6x$



5. $3y = x - 3$
 $x = 3$



6. $2x - y = 2$
 $y = x - 2$

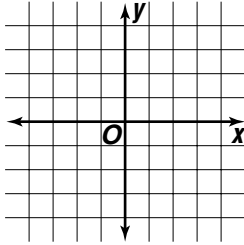


Practice

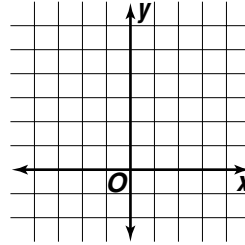
Families of Linear Graphs

Graph each pair of equations. Describe any similarities or differences and explain why they are a family of graphs.

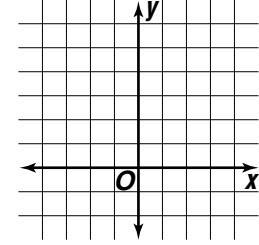
1. $y = 2x + 3$
 $y = 2x - 3$



2. $y = 4x + 5$
 $y = -3x + 5$

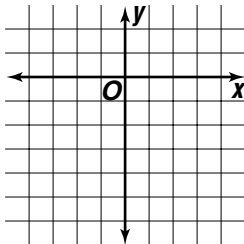


3. $y = \frac{1}{3}x + 2$
 $y = \frac{1}{3}x + 4$

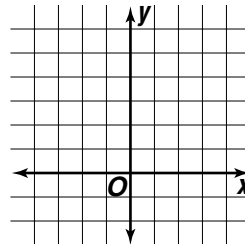


Compare and contrast the graphs of each pair of equations. Verify by graphing the equations.

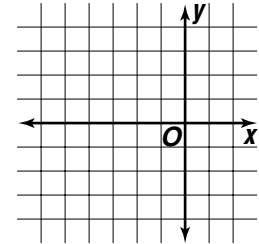
4. $y = -\frac{1}{2}x - 4$
 $y = -2x - 4$



5. $3x + 6 = y$
 $3x = y$



6. $y = \frac{5}{6}x + 3$
 $y = 5x + 3$



Change $y = -x + 2$ so that the graph of the new equation fits each description.

7. same slope,
shifted down 2 units

8. same y-intercept,
steeper negative slope

9. positive slope,
same y-intercept

10. same y-intercept, less
steep negative slope

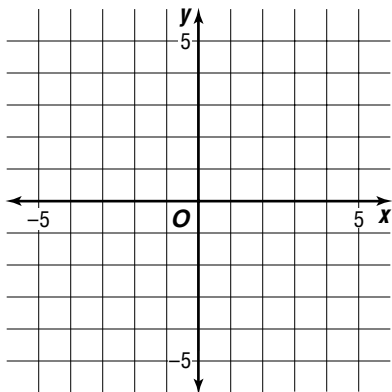
11. same slope, shifted
up 4 units

12. same slope, shifted
down 6 units

Enrichment

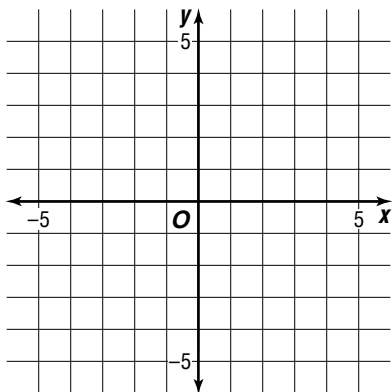
Inverse Relations

On each grid below, plot the points in Sets A and B. Then connect the points in Set A with the corresponding points in Set B. Then find the inverses of Set A and Set B, plot the two sets, and connect those points.



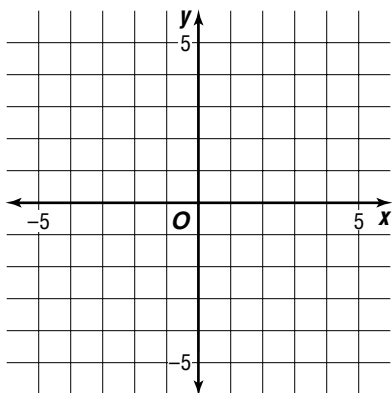
Set A	Set B
$(-4, 0)$	$(0, 1)$
$(-3, 0)$	$(0, 2)$
$(-2, 0)$	$(0, 3)$
$(-1, 0)$	$(0, 4)$

Inverse	
Set A	Set B
1. _____	_____
2. _____	_____
3. _____	_____
4. _____	_____



Set A	Set B
$(-3, -3)$	$(-2, 1)$
$(-2, -2)$	$(-1, 2)$
$(-1, -1)$	$(0, 3)$
$(0, 0)$	$(1, 4)$

Inverse	
Set A	Set B
5. _____	_____
6. _____	_____
7. _____	_____
8. _____	_____



Set A	Set B
$(-4, 1)$	$(3, 2)$
$(-3, 2)$	$(3, 2)$
$(-2, 3)$	$(3, 2)$
$(-1, 4)$	$(3, 2)$

Inverse	
Set A	Set B
9. _____	_____
10. _____	_____
11. _____	_____
12. _____	_____

13. What is the graphical relationship between the line segments you drew connecting points in Sets A and B and the line segments connecting points in the inverses of those two sets?

Fixed and Marginal Costs (Cost Control Analyst)

A cost control analyzer can estimate the *fixed cost* of production, that is, the cost incurred whether any product is manufactured. The analyzer can also estimate the cost of producing each additional item. This cost is called the *marginal cost*. These two costs combine to indicate the total cost t of producing x units.

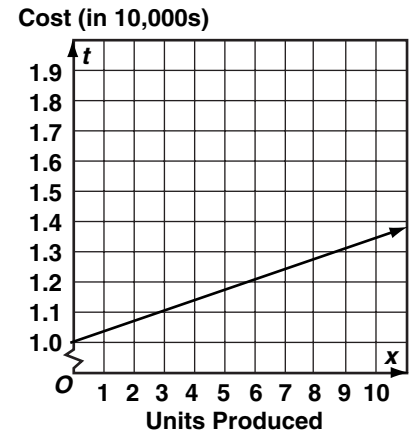
$$t = (\text{marginal cost})(x) + \text{fixed cost}$$

In the equation above, the marginal cost is the slope of the equation and the fixed cost is the y -intercept of the equation.

If a manufacturer of fire doors sets up an assembly that costs \$10,000 to operate and each fire door costs \$300 to make, describe the total cost of production t in terms of the number x of fire doors manufactured. Then graph the equation.

$$t = 300x + 10,000$$

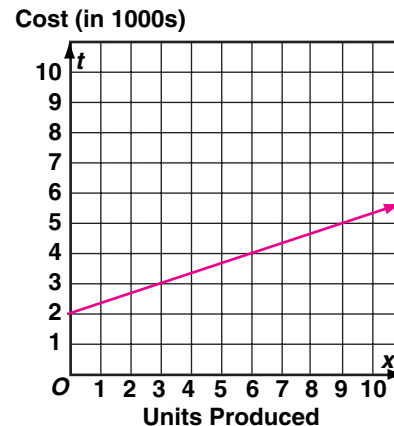
The graph is shown at the right.



Given the fixed and marginal costs, (a) find an equation for the total cost of production and (b) sketch a graph of the cost equation.

1. fixed cost: \$5000
marginal cost: \$250

2. fixed cost: \$2000
marginal cost: \$300



3. Use the equation in the example to find the cost of manufacturing 240 fire doors.