Study Guide

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Powers and Exponents

The product of a number with itself is called a **perfect square**. For example, 64 is a perfect square because $64 = 8 \times 8$. The **exponent** 2 can be used to write 8×8 as 8^2. Likewise, the exponent 4 can be used to write $8 \times 8 \times 8 \times 8$ as 8^4. Numbers that are expressed using exponents are called **powers**.

$$\text{base} \longrightarrow 8^4 \longleftarrow \text{exponent}$$

Example 1: Write $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ using exponents.
The base is 3. There are 5 factors of 3, so the exponent is 5.
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

Example 2: Write $x \cdot x \cdot x \cdot x$ using exponents.
The base is x. There are 4 factors of x, so the exponent is 4.
$x \cdot x \cdot x \cdot x = x^4$

Example 3: Write $5x^2y^3$ as a multiplication expression.
There is 1 factor of 5, 2 factors of x, and 3 factors of y.
$5 \cdot x \cdot x \cdot y \cdot y \cdot y$

Write each expression using exponents.

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

2. $(-3)(-3)(-3)(-3)$

3. $x \cdot x \cdot x \cdot x \cdot x$

4. $(-2)\ a \cdot a \cdot a \cdot a \cdot b$

5. $10 \cdot 10 \cdot 10 \cdot 10$

6. $5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

Write each power as a multiplication expression.

7. 7^3

8. $-5y^4$

9. d^4e^3

10. $9ab^3$

11. wx^2y^3

12. $(-2)^5$

Evaluate each expression if x = −3, y = 2, and z = −1.

13. zx^3y^2

14. $y(x^2 - z)$

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Powers and Exponents

Write each expression using exponents.

1. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

2. 8

3. $10 \cdot 10 \cdot 10 \cdot 10$

4. $7 \cdot 7 \cdot 7$

5. $(-4) \cdot (-4) \cdot (-4) \cdot (-4)$

6. $b \cdot b \cdot b \cdot b \cdot b \cdot b$

7. $x \cdot x$

8. $m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m$

9. $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$

10. $a \cdot a \cdot a \cdot a \cdot c \cdot c \cdot c \cdot c$

11. $7 \cdot 7 \cdot 9 \cdot 7 \cdot 9 \cdot 2 \cdot 2 \cdot 2$

12. $(6)(x)(x)(x)(y)(y)(y)(y)$

Write each power as a multiplication expression.

13. 9^3

14. 13^5

15. 7^2

16. p^4

17. n^6

18. $(-5)^5$

19. $4 \cdot 8^6$

20. $7^3 \cdot 5^2$

21. ab^2

22. m^5n^3

23. $-4c^3$

24. $3x^2y^4$

Evaluate each expression if a = −1, b = 3, and c = 2.

25. b^4

26. a^6

27. $4c^5$

28. $-3b^3$

29. a^5b^2

30. $2bc^3$

31. $-4a^4c^2$

32. $a^2 + b^2$

33. $2(b^2 - c^3)$

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Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a.	b.	c.
2^{10} = ________	5^{10} = ________	4^{10} = ________
2^9 = ________	5^9 = ________	4^9 = ________
2^8 = ________	5^8 = ________	4^8 = ________
2^7 = ________	5^7 = ________	4^7 = ________
2^6 = ________	5^6 = ________	4^6 = ________
2^5 = ________	5^5 = ________	4^5 = ________
2^4 = ________	5^4 = ________	4^4 = ________
2^3 = ________	5^3 = ________	4^3 = ________
2^2 = ________	5^2 = ________	4^2 = ________
2^1 = ________	5^1 = ________	4^1 = ________

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.

2. Describe the pattern of the powers from the top of the column to the bottom.

3. What would you expect the following powers to be?

2^0 5^0 4^0

4. Write a rule. Test it on patterns that you obtain using -2, -5, and -4 as bases.

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$ __?__ 0^{-1} does not exist.
0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?

6. Based upon the pattern, can you determine whether 0^0 exists?

7. The symbol 0^0 is called an **indeterminate,** which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?

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Multiplying and Dividing Powers

You can multiply powers by using the following rules.

Product of Powers	Examples
To multiply powers with the same base, add the exponents. $a^m \cdot a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4}$ or 3^6 $x^3 \cdot x^2 = x^{3+2}$ or x^5 $(ab^2) \cdot (a^3b^4) = (a \cdot a^3)(b^2 \cdot b^4)$ or a^4b^6

You can divide powers by using the following rules.

Quotient of Powers	Examples
To divide powers with the same base, subtract the exponents. $\frac{a^m}{a^n} = a^{m-n}$	$\frac{10^4}{10^3} = 10^{4-3}$ or 10^1 $\frac{x^5}{x^2} = x^{5-2}$ or x^3 $\frac{b^8c^4}{b^2c} = b^{8-2} \cdot c^{4-1}$ or b^6c^3

Simplify each expression.

1. $6^3 \cdot 6^2$

2. $(-3)\ x^2 \cdot x^3$

3. $y^8 \cdot y^9$

4. $(5m^3)(3m^2n^4)$

5. $(ab^2)(a^2b^3)$

6. $(-10x^4y^2)(3x^2y)$

7. $\frac{8^9}{8^7}$

8. $\frac{w^6}{w^3}$

9. $\frac{a^7b^2}{a^6b}$

10. $\frac{9j^5}{3j^2}$

11. $\frac{36x^9y^5}{18x^3y^2}$

12. $\frac{-24m^6n^3}{6m^2n^2}$

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Multiplying and Dividing Powers

Simplify each expression.

1. $6^3 \cdot 6^2$

2. $7^6 \cdot 7^4$

3. $y^4 \cdot y^8$

4. $b \cdot b^4$

5. $(g^2)(g^3)(g)$

6. $m(m^8)$

7. $(a^2b^3)(a^4b)$

8. $(xy^5)(x^3y^3)$

9. $(2c^3)(2c)$

10. $(-3x^2)(6x^2)$

11. $(-7xy)(-2x)$

12. $(5m^3n^2)(4m^2n^3)$

13. $(-8ab)(a^2b^5)$

14. $\frac{9^2}{9}$

15. $\frac{12^8}{12^3}$

16. $\frac{y^4}{y^2}$

17. $\frac{k^6}{k^6}$

18. $\frac{x^4y^5}{x^3y^2}$

19. $\frac{a^9b^6}{a^2b}$

20. $\frac{mn^3}{n^2}$

21. $\frac{15a^3}{3a}$

22. $\frac{8x^5y^4}{4x^2y^2}$

23. $\frac{m^2n}{m^2}$

24. $\frac{6a^5b^7}{-2a^3b^7}$

25. $\frac{-20x^3y^2}{-5x^3y}$

26. $\frac{-16ab^4}{4b^3}$

27. $\frac{12x^2y}{2x^2y}$

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The Four Digits Problem

One well-known problem in mathematics is to write expressions for consecutive numbers from 1 upward as far as possible. On this page, you will use the digits 1, 2, 3, and 4. Each digit is used only once. You can use addition, subtraction, multiplication (not division), exponents, and parentheses in any way you wish. Also, you can use two digits to make one number, as in 12 or 34.

Express each number as a combination of the digits 1, 2, 3, and 4.

$1 = (3 \times 1) - (4 - 2)$

2 = ______

3 = ______

4 = ______

5 = ______

6 = ______

7 = ______

8 = ______

9 = ______

10 = ______

11 = ______

12 = ______

13 = ______

14 = ______

15 = ______

16 = ______

17 = ______

18 = ______

$19 = 3(2 + 4) + 1$

20 = ______

21 = ______

22 = ______

$23 = 31 - (4 \times 2)$

24 = ______

25 = ______

26 = ______

27 = ______

28 = ______

29 = ______

30 = ______

31 = ______

32 = ______

33 = ______

34 = ______

$35 = 2^{(4+1)} + 3$

36 = ______

37 = ______

38 = ______

39 = ______

40 = ______

41 = ______

42 = ______

$43 = 42 + 1^3$

44 = ______

45 = ______

46 = ______

47 = ______

48 = ______

49 = ______

50 = ______

Does a calculator help in solving these types of puzzles? Give reasons for your opinion.

____

____

____

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Negative Exponents

In science and technology, negative exponents are sometimes used to represent very small numbers. For example, the diameter of an atom is expressed as 10^{-10} meter. This is the decimal 0.0000000001. This number expressed as a fraction is $\frac{1}{10,000,000,000}$.

When simplifying an expression with a negative exponent, you may need to use the Quotient of Powers rule.

Negative Exponents	Examples
$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2}$ or $\frac{1}{9}$ $x^{-3} = \frac{1}{x^3}$ $\frac{b^2}{b^{-3}} = b^{2-(-3)}$ or b^5

Remember that a negative exponent is used to write a reciprocal, not to represent a negative number.

Simplify each expression.

1. 5^{-3}

2. 3^{-2}

3. y^{-8}

4. $5m^{-3}$

5. $(a^{-2})(b^3)$

6. $-6x^{-4}y^6$

7. $\frac{3^4}{3^{-2}}$

8. $\frac{k^{-3}}{k^5}$

9. $\frac{12x^5}{4x^{-2}}$

10. $a^2b^{-2}c^{-1}$

11. $\frac{-24m^6n^3}{3m^2n}$

12. $\frac{36x^9y^5z^{-2}}{9x^3y^2}$

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Negative Exponents

Write each expression using positive exponents. Then evaluate the expression.

1. 2^{-6}

2. 5^{-1}

3. 8^{-2}

4. 10^{-3}

Simplify each expression.

5. g^{-6}

6. s^{-1}

7. q^{0}

8. $a^{-2}b^{2}$

9. $m^{5}n^{-1}$

10. $p^{-1}q^{-6}r^{3}$

11. $x^{-3}y^{2}z^{-4}$

12. $a^{-2}b^{0}c^{-1}$

13. $12m^{-6}n^{4}$

14. $7xy^{-8}z$

15. $x^{-3}(x^{2})$

16. $b^{3}(b^{-5})$

17. $\frac{b^{3}}{b^{6}}$

18. $\frac{y^{3}}{y^{-2}}$

19. $\frac{m^{5}n^{3}}{m^{6}n^{2}}$

20. $\frac{xy^{2}}{xy^{3}}$

21. $\frac{a^{7}b^{4}}{a^{9}b^{2}}$

22. $\frac{rs^{-3}}{r^{2}s^{4}}$

23. $\frac{16c^{8}}{4c^{10}}$

24. $\frac{9x^{-5}y^{5}}{36x^{4}y^{3}}$

25. $\frac{7p^{2}q^{6}}{21p^{-3}q^{7}}$

26. $\frac{-6m^{5}n^{2}q^{-1}}{36m^{-2}n^{4}q^{-1}}$

27. $\frac{4a^{3}b^{2}c^{2}}{6a^{5}b^{3}c}$

28. $\frac{28x^{5}y^{-3}z}{-4x^{4}yz^{3}}$

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Rational Exponents

You have developed the following properties of powers when a is a positive real number and m and n are integers.

$a^m \cdot a^n = a^{m+n}$	$(ab)^m = a^m b^m$	$a^0 = 1$
$(a^m)^n = a^{mn}$	$\frac{a^m}{a^n} = a^{m-n}$	$a^{-m} = \frac{1}{a^m}$

Exponents need not be restricted to integers. We can define rational exponents so that operations involving them will be governed by the properties for integer exponents.

$\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2}\cdot 2} = a$	$\left(a^{\frac{1}{3}}\right)^3 = a^{\frac{1}{3}\cdot 3}$	$\left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n}\cdot n} = a$
$a^{\frac{1}{2}}$ squared is a.	$a^{\frac{1}{3}}$ cubed is a.	$a^{\frac{1}{n}}$ to the n power is a.
$a^{\frac{1}{2}}$ is a square root of a.	$a^{\frac{1}{3}}$ is a cube root of a.	$a^{\frac{1}{n}}$ is an nth root of a.
$a^{\frac{1}{2}} = \sqrt{a}$	$a^{\frac{1}{3}} = \sqrt[3]{a}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$

Now let us investigate the meaning of $a^{\frac{m}{n}}$.

$a^{\frac{m}{n}} = a^{m \cdot \frac{1}{n}}(a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$ $\qquad a^{\frac{m}{n}} = a^{\frac{1}{n}\cdot m} = \left(a^{\frac{1}{n}}\right)^m = (\sqrt[n]{a})^m$

Therefore, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$.

Example 1: Write $\sqrt[4]{a^3}$ in exponential form.

$\sqrt[4]{a^3} = a^{\frac{3}{4}}$

Example 2: Write $a^{\frac{2}{5}}$ in radical form.

$a^{\frac{2}{5}} = \sqrt[5]{a^2}$

Example 3: Find $\frac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}}$.

$$\frac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}} = a^{\frac{2}{3}-\frac{1}{2}} = a^{\frac{4}{6}-\frac{3}{6}} = a^{\frac{1}{6}} \text{ or } \sqrt[6]{a}$$

Write each expression in radical form.

1. $b^{\frac{3}{2}}$ **2.** $3c^{\frac{1}{2}}$ **3.** $(3c)^{\frac{1}{2}}$

Write each expression in exponential form.

4. $\sqrt[3]{b^4}$ **5.** $\sqrt{4a^3}$ **6.** $2 \cdot \sqrt[3]{b^2}$

Perform the operation indicated. Answers should show positive exponents only.

7. $\left(a^3 b^{\frac{1}{4}}\right)^2$ **8.** $\frac{-8a^{\frac{3}{4}}}{2a^{\frac{1}{2}}}$ **9.** $\left(\frac{b^{\frac{1}{2}}}{b^{-\frac{2}{3}}}\right)^3$

10. $\sqrt{a^3} \cdot \sqrt{a}$ **11.** $\left(a^2 b^{-\frac{1}{3}}\right)^{-\frac{1}{2}}$ **12.** $-2a^{\frac{1}{3}}b^0\left(5a^{\frac{1}{2}}b^{-\frac{2}{3}}\right)$

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Scientific Notation

In science and in other applications, **scientific notation** is often used to represent very small or very large numbers. For example, the speed of light is about 3×10^8 meters per second. This represents the number 300,000,000 meters per second.

In scientific notation, a number is expressed in the form $a \times 10^n$, where a is a number greater than 1 and less than 10 and n is an integer.

Scientific Notation	Examples
1. Place the decimal point after the first non-zero digit in the given number. **2.** Find the power of 10 by counting decimal places. When the given number is one or greater, the power of 10 is positive. When the given number is between zero and one, the exponent of 10 is negative.	$6{,}200{,}000 = 6.2 \times 10^6$ $0.000056 = 5.6 \times 10^{-5}$

Express each number in standard form.

1. 6.1×10^4

2. 4.8×10^{-2}

3. 8.12×10^3

4 5×10^7

5. 9×10^{-5}

6. 1.1×10^{-7}

7. 2.15×10^5

8. 5.1651×10^3

Express each number in scientific notation.

9. 8400

10. 3,000,000

11. 0.05

12. 14.2

13. 0.00048

14. 82,000,000,000

15. 0.0000725

16. 6

Practice

Scientific Notation

Express each measure in standard form.

1. 4 gigabytes

2. 78 kilowatts

3. 9 megahertz

4. 7.5 milliamperes

5. 2.3 nanoseconds

6. 3.7 micrograms

Express each number in scientific notation.

7. 6300

8. 4,600,000

9. 92.3

10. 51,200

11. 776,000

12. 68,200,000

13. 0.00013

14. 0.000009

15. 0.026

16. 0.04

17. 0.0055

18. 0.000031

Evaluate each expression. Express each result in scientific notation and in standard form.

19. $(4 \times 10^3)(2 \times 10^4)$

20. $(3 \times 10^2)(1.5 \times 10^{-5})$

21. $(6 \times 10^{-7})(1.5 \times 10^9)$

22. $(7 \times 10^{-3})(2.1 \times 10^{-3})$

23. $\frac{5.1 \times 10^5}{1.7 \times 10^7}$

24. $\frac{3.6 \times 10^6}{2 \times 10^2}$

25. $\frac{8.5 \times 10^{-3}}{2.5 \times 10^6}$

26. $\frac{2.7 \times 10^2}{3 \times 10^{-4}}$

27. $\frac{3.9 \times 10^4}{3 \times 10^7}$

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Converting Metric Units

Scientific notation is convenient to use for unit conversions in the metric system.

Example 1: How many kilometers are there in 4,300,000 meters?

Divide the measure by the number of meters (1000) in one kilometer. Express both numbers in scientific notation.

$\frac{4.3 \times 10^6}{1 \times 10^3} = 4.3 \times 10^3$ The answer is 4.3×10^3 km.

Example 2: Convert 3700 grams into milligrams.

Multiply by the number of milligrams (1000) in 1 gram.

$(3.7 \times 10^3)(1 \times 10^3) = 3.7 \times 10^6$ There are 3.7×10^6 mg in 3700 g.

Complete the following. Express each answer in scientific notation.

1. 250,000 m = ______________ km

2. 375 km = ______________ m

3. 247 m = ______________ cm

4. 5000 m = ______________ mm

5. 0.0004 km = ______________ m

6. 0.01 mm = ______________ m

7. 6000 m = ______________ mm

8. 340 cm = ______________ km

9. 52,000 mg = ______________ g

10. 420 kL = ______________ L

Solve.

11. The planet Mars has a diameter of 6.76×10^3 km. What is the diameter of Mars in meters? Express the answer in both scientific and decimal notation.

12. The distance of the earth from the sun is 149,590,000 km. Light travels 3.0×10^8 meters per second. How long does it take light from the sun to reach the earth in minutes?

13. A light-year is the distance that light travels in one year. (See Exercise 12.) How far is a light year in kilometers? Express your answer in scientific notation.

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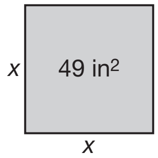
Square Roots

Suppose you know that the area of the square below is 49 square inches. What is the length of a side? Since $7 \times 7 = 49$, each side is 7 inches long.



Also, since $7 \times 7 = 49$, we say that the **square root** of 49 is 7. A shorter way to write this is with the symbol $\sqrt{}$, a **radical sign**. Write $\sqrt{49} = 7$. Use the following rules to simplify square roots.

Square Roots	Examples
1. The square root of a number is one of its equal factors.	$\sqrt{49} = 7$
2. The square root of a product is equal to the product of each square root. $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt{225} = \sqrt{9} \cdot \sqrt{25} = 3 \cdot 5 = 15$
3. The square root of a quotient is equal to the quotient of each square root. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{25}{144}} = \frac{\sqrt{25}}{\sqrt{144}} = \frac{5}{12}$

Simplify.

1. $\sqrt{81}$

2. $-\sqrt{25}$

3. $\sqrt{100}$

4. $-\sqrt{400}$

5. $\sqrt{625}$

6. $-\sqrt{1156}$

7. $\sqrt{\frac{81}{49}}$

8. $\sqrt{\frac{25}{36}}$

9. $-\sqrt{\frac{16}{100}}$

10. $\sqrt{0.25}$

11. $-\sqrt{0.0016}$

12. $-\sqrt{\frac{0.16}{0.09}}$

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Square Roots

Simplify.

1. $\sqrt{36}$

2. $-\sqrt{16}$

3. $\sqrt{81}$

4. $-\sqrt{144}$

5. $-\sqrt{100}$

6. $-\sqrt{121}$

7. $\sqrt{169}$

8. $-\sqrt{25}$

9. $-\sqrt{529}$

10. $\sqrt{256}$

11. $\sqrt{324}$

12. $-\sqrt{289}$

13. $\sqrt{441}$

14. $-\sqrt{225}$

15. $\sqrt{196}$

16. $\sqrt{400}$

17. $\sqrt{484}$

18. $\sqrt{729}$

19. $-\sqrt{625}$

20. $\sqrt{1225}$

21. $\sqrt{\frac{49}{81}}$

22. $-\sqrt{\frac{16}{25}}$

23. $\sqrt{\frac{4}{16}}$

24. $-\sqrt{\frac{25}{36}}$

25. $-\sqrt{\frac{100}{121}}$

26. $\sqrt{\frac{1}{64}}$

27. $\sqrt{\frac{36}{64}}$

28. $-\sqrt{\frac{144}{36}}$

29. $-\sqrt{\frac{121}{289}}$

30. $-\sqrt{\frac{225}{625}}$

31. $\sqrt{\frac{400}{100}}$

32. $\sqrt{\frac{196}{256}}$

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Standard Deviation

The most commonly used measure of variation is called the **standard deviation.** It shows how far the data are from their mean. You can find the standard deviation using the steps given below.

a. Find the mean of the data.
b. Find the difference between each value and the mean.
c. Square each difference.
d. Find the mean of the squared differences.
e. Find the square root of the mean found in Step d. The result is the standard deviation.

Example: Calculate the standard deviation of the test scores 82, 71, 63, 78, and 66.

$$\text{mean of the data } (m) = \frac{82 + 71 + 63 + 78 + 66}{5} = \frac{360}{5} = 72$$

x	$x - m$	$(x - m)^2$
82	$82 - 72 = 10$	$10^2 = 100$
71	$71 - 72 = -1$	$(-1)^2 = 1$
63	$63 - 72 = -9$	$(-9)^2 = 81$
78	$78 - 72 = 6$	$6^2 = 36$
66	$66 - 72 = -6$	$(-6)^2 = 36$

$$\text{mean of the squared differences} = \frac{100 + 1 + 81 + 36 + 36}{5} = \frac{254}{5} = 50.8$$

$$\text{standard deviation} = \sqrt{50.8} \approx 7.13$$

Use the test scores 94, 48, 83, 61, and 74 to complete Exercises 1–3.

1. Find the mean of the scores.

2. Show that the standard deviation of the scores is about 16.2.

3. Which had less variations, the test scores listed above or the test scores in the example?

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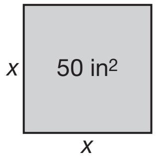
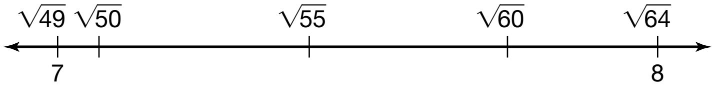
Estimating Square Roots

Suppose you know that the area of the square below is 50 square inches. What is the length of a side?



Since there is no rational number whose square is 50, you need to estimate the answer. If you use a calculator to find $\sqrt{50}$, it will return an approximate value of 7.071067812. This represents an **irrational number**, a decimal number that does not repeat or terminate.

You can use perfect squares to estimate irrational square roots. Since 50 is close to 49, $\sqrt{50} \approx 7$, so the length of the side of the square is about 7 inches. Likewise, if the area of a square is 60 square inches, the side length would be $\sqrt{60} \approx 8$ inches, since the actual value is close to $\sqrt{64}$, or 8.



Estimate each square root to the nearest whole number.

1. $\sqrt{90}$

2. $\sqrt{134}$

3. $\sqrt{17}$

4. $\sqrt{500}$

5. $\sqrt{1000}$

6. $\sqrt{98}$

7. $\sqrt{320}$

8. $\sqrt{5}$

9. $\sqrt{75}$

10. $\sqrt{84.5}$

11. $\sqrt{128.9}$

12. $\sqrt{0.025}$

13. $\sqrt{0.0075}$

14. $\sqrt{10.01}$

15. $\sqrt{0.9988}$

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Practice

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Estimating Square Roots

Estimate each square root to the nearest whole number.

1. $\sqrt{10}$ **2.** $\sqrt{14}$ **3.** $\sqrt{32}$

4. $\sqrt{19}$ **5.** $\sqrt{40}$ **6.** $\sqrt{6}$

7. $\sqrt{53}$ **8.** $\sqrt{23}$ **9.** $\sqrt{30}$

10. $\sqrt{21}$ **11.** $\sqrt{90}$ **12.** $\sqrt{73}$

13. $\sqrt{72}$ **14.** $\sqrt{56}$ **15.** $\sqrt{89}$

16. $\sqrt{135}$ **17.** $\sqrt{152}$ **18.** $\sqrt{110}$

19. $\sqrt{162}$ **20.** $\sqrt{129}$ **21.** $\sqrt{181}$

22. $\sqrt{174}$ **23.** $\sqrt{223}$ **24.** $\sqrt{195}$

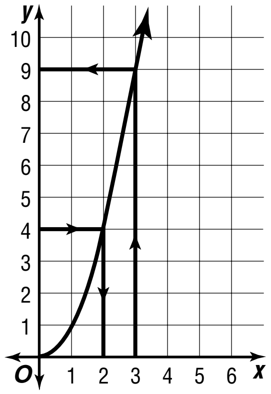
25. $\sqrt{240}$ **26.** $\sqrt{271}$ **27.** $\sqrt{312}$

28. $\sqrt{380}$ **29.** $\sqrt{335}$ **30.** $\sqrt{300}$

Enrichment

Squares and Square Roots From a Graph

The graph of $y = x^2$ can be used to find the squares and square roots of numbers.



To find the square of 3, locate 3 on the x-axis. Then find its corresponding value on the y-axis.

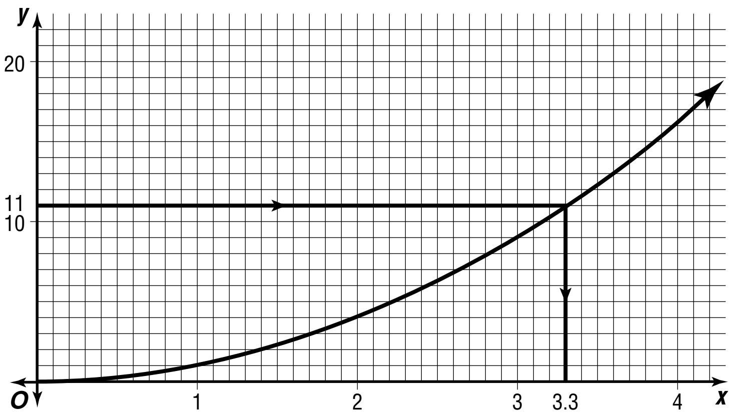
The arrows show that $3^2 = 9$.

To find the square root of 4, first locate 4 on the y-axis. Then find its corresponding value on the x-axis. Following the arrows on the graph, you can see that $\sqrt{4} = 2$.

A small part of the graph at $y = x^2$ is shown below. A 1:10 ratio for unit length on the y-axis to unit length on the x-axis is used.

Example: Find $\sqrt{11}$.

The arrows show that $\sqrt{11} = 3.3$ to the nearest tenth.



Use the graph above to find each of the following to the nearest whole number.

1. 1.5^2 **2.** 2.7^2 **3.** 0.9^2

4. 3.6^2 **5.** 4.2^2 **6.** 3.9^2

Use the graph above to find each of the following to the nearest tenth.

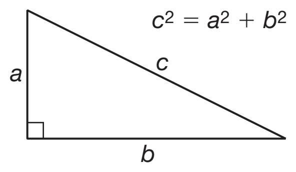
7. $\sqrt{15}$ **8.** $\sqrt{8}$ **9.** $\sqrt{3}$

10. $\sqrt{5}$ **11.** $\sqrt{14}$ **12.** $\sqrt{17}$

Study Guide

The Pythagorean Theorem

One of the most famous theorems in mathematics gives the relationship among the sides of a right triangle. The Pythagorean Theorem states that the square of the length of the **hypotenuse** of a right triangle is equal to the sum of the squares of the lengths of the other two sides, also known as **legs**. Use the model shown for a right triangle with hypotenuse c.



This relationship is powerful because it is true for *any* right triangle.

Example 1: Find the length of the hypotenuse of a right triangle with side lengths 5 centimeters and 12 centimeters.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144 = 169$$
$$c = \sqrt{169}$$
$$c = 13$$

The length of the hypotenuse is 13 centimeters.

Example 2: Find the length of a side of a right triangle if the length of the hypotenuse is 25 feet and the length of one leg is 7 feet.

$$c^2 = a^2 + b^2$$
$$25^2 = 7^2 + b^2$$
$$625 = 49 + b^2$$
$$625 - 49 = 49 + b^2 - 49$$
$$576 = b^2$$
$$b = \sqrt{576}$$
$$b = 24$$

The length of the leg is 24 feet.

If c is the measure of the hypotenuse of a right triangle and a and b are the measures of the legs, find each missing measure.

1. $a = 3$, $b = 4$, $c = ?$ **2.** $a = 7$, $b = 24$, $c = ?$ **3.** $a = 9$, $b = 40$, $c = ?$

4. $a = 5$, $b = ?$, $c = 13$ **5.** $a = 20$, $b = ?$, $c = 29$ **6.** $a = ?$, $b = 8$, $c = 10$

7. $a = ?$, $b = 48$, $c = 50$ **8.** $a = 9$, $b = ?$, $c = 15$ **9.** $a = 60$, $b = 80$, $c = ?$

10. $a = ?$, $b = 36$, $c = 45$ **11.** $a = 40$, $b = 42$, $c = ?$ **12.** $a = 25$, $b = ?$, $c = 65$

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Enrichment

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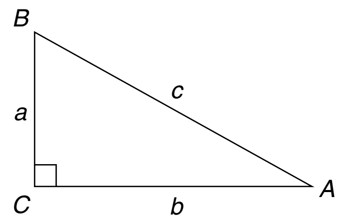
Pythagorean Triples

Recall the Pythagorean Theorem.
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$a^2 + b^2 = c^2$
Note that c is the length of the hypotenuse.

The integers 3, 4, and 5 satisfy the Pythagorean Theorem and can be the lengths of the sides of a right triangle.

$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

Furthermore, for any positive integer n, the numbers $3n$, $4n$, and $5n$ satisfy the Pythagorean Theorem.

For $n = 2$: $6^2 + 8^2 = 10^2$
$36 + 64 = 100$
$100 = 100$

If three numbers satisfy the Pythagorean Theorem, they are called a **Pythagorean triple.** Here is an easy way to find other Pythagorean triples.

The numbers a, b, and c are a Pythagorean triple if
$a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$,
where m and n are relatively prime positive integers and $m > n$.

Example: Choose $m = 5$ and $n = 2$.

$a = m^2 - n^2 = 5^2 - 2^2 = 25 - 4 = 21$

$b = 2mn = 2(5)(2) = 20$

$c = m^2 + n^2 = 5^2 + 2^2 = 25 + 4 = 29$

Check: $20^2 + 21^2 = 29^2$
$400 + 441 = 841$
$841 = 841$

Use the following values of m and n to find Pythagorean triples.

1. $m = 3$ and $n = 2$

2. $m = 4$ and $n = 1$

3. $m = 5$ and $n = 3$

4. $m = 6$ and $n = 5$

5. $m = 10$ and $n = 7$

6. $m = 8$ and $n = 5$

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School-to-Workplace

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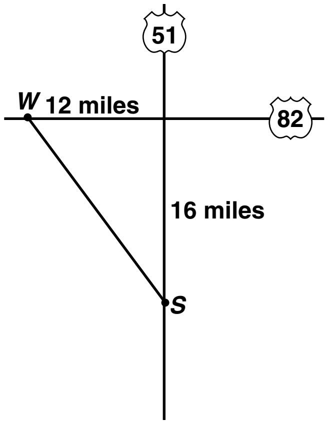
Roadways (Concrete Contractor)

Homeowners, business owners, school administrators, and city managers want sidewalks and roadways made of different kinds of material. Concrete is one of the most popular and serviceable. A concrete contractor is the one who mixes, delivers, and pours the concrete. First, however, the contractor must get some information about the geometry of the land.

Routes 82 and 51 intersect in the center of Pine City. Route 82 runs east and west and Route 51 runs north and south. To reduce the traffic in the center of town, government officials plan to build a road connecting a point 16 miles south of the center of town on Route 51 to a point 12 miles west of the center of town on Route 82. Before a contractor can give an estimate for the cost of the job, she must determine the length of the new road. Since the roads form a right triangle, the Pythagorean Theorem can be used to find the length of the new road.



$$c^2 = a^2 + b^2$$
$$c^2 = 16^2 + 12^2$$
$$c^2 = 256 + 144$$
$$c^2 = 400$$
$$c = 20$$

The new road will be 20 miles long.

Solve.

1. Suppose a person wishes to get from point S to point W, how many less miles will the person travel by taking the new road instead of Routes 51 and 82?

2. If the new road is built from a point 14 miles south of the center of town to a point 10 miles west of the center of town, what is the length of the new road?

3. How much shorter would the road in Exercise 2 be than the road in the example?