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Polynomials

The expressions y, -6x, $5a^2$, and $10cd^3$ are examples of **monomials**. A monomial is a number, a variable, or a product of numbers and variables. Any exponents in a monomial are positive integers. The exponents cannot be variables.

Example: Which of the following expressions are monomials?

 $-8g \qquad 2z^{-4} \qquad 17t^5v \qquad 3-a \qquad \frac{9}{pq} \qquad 6^y$

-8g and $17t^5v$ are monomials because they are products of numbers and variables.

 $2z^{-4}$ is not a monomial because it has a negative exponent.

3 - a is not a monomial because it involves subtraction.

 $\frac{9}{pq}$ is not a monomial because it involves division.

 6^{y} is not a monomial because its exponent is a variable.

The sum of two or more monomials is called a **polynomial**. Each monomial is a term of the polynomial. Polynomials with two or three terms have special names.

 $15r^4 + 1$ is a **binomial**. It has two terms, $15r^4$ and 1. $-9 + g - 4g^2$ is a **trinomial**. It has three terms, -9, g, and $-4g^2$.

Determine whether each expression is a monomial. Explain why or why not.

 1. 18x + 2 2. $-21s^4t^2$

 3. w^{-2} 4. $\frac{4}{5}a^3b$

State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, or trinomial.

5. $\frac{8}{x}$	6. $-7r + 9s - 3$
7. $abc^3 - a^3bc$	8. $35u^5v^6$
9. $5 + 5^k$	10. $8d - 9e \div f$
11. $16x - 16y$	12. $8j^2 + 3j - 7$
13. $3m^3 + \frac{1}{3}m$	14. $-14p + p^{-14}$

Practice

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Polynomials

Determine whether each expression is a monomial. Explain why or why not.

1. $8y^2$	2. $3m^{-4}$
3. $\frac{6}{p}$	49
5. $2x^2 + 5$	6. $-7a^{3}b$

State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, or trinomial.

7. $4h + 8$	8. 13	9. 3 <i>xy</i>
10. $\frac{2}{c} + 4$	11. $m^2 + 2 - m$	12. $5a + b^{-2}$
13. $7 - \frac{1}{2}d$	14. n^2	15. $2a^2 + 8a + 9 - 3$
16. $x^3 + 4x^3$	17. $m^2 + 2mn + n^2$	18. 6 – <i>y</i>
Find the degree of eac	h polynomial.	
19. 8	20. $3a^2$	21. $5m + n^2$
22. 16 <i>cd</i>	23. $3g^4 + 2h^3$	24. $4a^2b + 3ab^3$
25. $c^2 + 2c - 8$	26. $2p^3 - 7p^2 - 4p$	27. $9y^3z + 15y^5z$
28. $7s^2 - 4s^2t + 2st$	29. $6x^3 + x^3y^2 - 3$	30. $2ab^3 - 5abc$

Enrichment

Polynomials and Volume

The volume of a rectangular prism can be written as the product of three polynomials. Recall that the volume equals the length times the width times the height.

The two volumes at the right represent the cube of *y* and the cube of *x*.

Multiply to find the volume of each prism. Write each answer as an algebraic expression.



Multiply, then add to find each volume. Write the answer as an algebraic expression.



x

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x

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Adding and Subtracting Polynomials

To add polynomials, group the like terms together and then find the sum. $3x^2$ and x^2 are like terms. x^2 and x, and x^2 and y^2 are unlike terms.

Example 1: Find
$$(3x^2 + 2x - 5) + (x^2 + 4x + 4)$$
.
 $(3x^2 + 2x - 5) + (x^2 + 4x + 4)$
 $= (3x^2 + 2x + (-5)) + (x^2 + 4x + 4)$ Rewrite subtraction.
 $= (3x^2 + x^2) + (2x + 4x) + (-5 + 4)$ Regroup like terms.
 $= (3 + 1)x^2 + (2 + 4)x + (-5 + 4)$ Distributive property
 $= 4x^2 + 6x - 1$ Simplify.

You can subtract a polynomial by adding its additive inverse.

Example 2: Find the additive inverse of $5b^2 - 3$. The additive inverse is $-(5b^2 - 3)$ or $-5b^2 + 3$.

Example 3: Find
$$(4m^3 - 6) - (7m^3 - 9)$$
.
 $(4m^3 - 6) - (7m^3 - 9)$
 $= (4m^3 - 6) + (-7m^3 + 9)$ The additive inverse of $7m^3 - 9$ is $-7m^3 + 9$.
 $= (4m^3 - 7m^3) + (-6 + 9)$ Regroup like terms.
 $= (4 - 7)m^3 + (-6 + 9)$ Distributive property
 $= -3m^3 + 3$ Simplify.

Find each sum or difference.

1.
$$(2a + 3) + (5a + 1)$$
2. $(8w^2 + w) + (7w^2 - 3w)$ 3. $(-5c^4 + 2c^2 - 6) + (6c^4 - 2c^2 + 5)$ 4. $(12m - 5n) + (12m + 5n)$ 5. $(4g + h^3) + (-9g - 4h^3)$ 6. $(2 - 16x^2) + (8 - 16x^2)$ 7. $(18 + 5xy) + (-6 - 10xy)$ 8. $(35a^2 + 15a - 20) + (10a^2 + 25)$ 9. $(6d + 3) - (4d + 5)$ 10. $(14 - 3t) - (2 + 7t)$ 11. $(-18s^2 + s) - (6s^2 - 8s)$ 12. $(26g - 13gh) - (-2g + gh)$ 13. $(7y^2 + 2y + 21) - (9y^2 + 6y + 11)$ 14. $(-5m^2 + 2n - 1) - (7m^2 + 16n - 8)$

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Adding and Subtracting Polynomials

Practice

	_	-				
Find e	each sum.					
1. 	5x - 2 +) $4x + 6$	2. 2 (+)	$\frac{y+4}{y-1}$	3	$\begin{array}{c} 4x - 8 \\ (+) \ 2x + 5 \end{array}$	
4. <u>(</u> -	$2x^2 - 7x - 4 +) x^2 + 3x + 2$	5. <u>(+)</u>	$\frac{n^2+4n+3n^2+4n-1}{n^2+4n-1}$	3 6 4	$\frac{2x^2 + 3xy}{(+)\ 2x^2 - 2xy}$	$-y^2$ $-4y^2$
7. (2	$2x^2 - 2x - 4) + (x^2 - 5)$	3x + 2)	8.	$(x^2 + 2x +$	$1) + (3x^2 + 4x -$	+ 1)
9. (2	$2a^2 + 8a + 6) + (a^2 + 6)$	3a - 4)	10.	$(x^2 + x - 1)$	2) + $(x^2 - 3x)$	
11. (3	$8x^2 + 8x + 4) + (4x^2 - 4x^2)$	1)	12.	$(x^2 - 4x - 4x)$	$(5) + (x^2 + 4x)$	
Find o	each difference.					
13. (-	7n + 2 -) $n + 1$	14. <u>(-)</u>	$\frac{3x-3}{2x+2}$	15	2y + 5 (-) y - 1	
16. (-	$4x^2 + 7x - 2$ -) $2x^2 - 6x + 4$	17. 2 (-)	$2x^2 - 9x - 5x^2 - 5x - 6x^2 - 5x - 6x^2 - 5x - 6x^2 - 5x^2 - 6x^2 - 6$	18	$5m^2 - 4m$ (-) $4m^2 + 8m$	$\frac{2}{2} - 1}{\frac{2}{2} + 4}$
19. (6	(3x-2) - (8x+3)		20.	$(3x^2 + 3x -$	$(-6) - (2x^2 - 2x)$	- 4)
21. (6	$6x^2 + 2x - 8) - (4x^2 + 6x^2)$	8x + 4)	22.	$(2a^2 + 6a +$	$(-4) - (a^2 - 3)$	
23. (2	$2x^2 - 8x + 3) - (-x^2 +$	2x)	24.	$(3x^2 - 5xy)$	$(-2y^2) - (2x^2 +$	<i>y</i> ²)

Enrichment

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Geometric Series

The terms of this polynomial form a geometric series.

 $a + ar + ar^2 + ar^3 + ar^4$

The first term is the constant a. Then each term after that is found by multiplying by a constant multiplier r.

Use the equation $S = a + ar + ar^2 + ar^3 + ar^4$ for Exercises 1–3.

1. Multiply each side of the equation by r.

2. Subtract the original equation from your result in Exercise 1.

3. Solve the result from Exercise 2 for the variable *S*.

Use the polynomial $a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$ for Exercises 4–8.

- 4. Write the 10th term of the polynomial.
- **5.** If a = 5 and r = 2, what is the 8th term?
- **6.** Follow the steps in Exercises 1–3 to write a formula for the sum of this polynomial.
- 7. If the 3rd term is 20 and the 6th term is 160, solve for r^3 and then find r. Then solve $ar^2 = 20$ for a and find the value of the first six terms of the polynomial.
- 8. Find the sum of the first six terms of the geometric series that begins 3, 6, 12, 24, \cdots . First write the values for a and r.

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Multiplying a Polynomial by a Monomial

You can use the Distributive Property to multiply a polynomial by a monomial.

Example 1: Multiply 6(x + 7). 6(x + 7) = 6(x) + 6(7)= 6x + 42Distributive Property. Simplify. **Example 2:** Multiply $-2(g^2 + 3g - 5)$. $-2(g^2 + 3g - 5) = -2(g^2) + (-2)(3g) + (-2)(-5)$ = -2g^2 - 6g + 10 **Example 3:** Multiply 9a(a + 1). 9a(a + 1) = 9a(a) + 9a(1) $= 9a^{2} + 9a$

Some equations require that you multiply polynomials.

Example 4: Solve 5(x + 3) = 25.

5(x+3) = 25	
5x + 15 = 25	Distributive Property
5x + 15 - 15 = 25 - 15	Subtract 15 from each side.
5x = 10	Combine like terms.
x = 2	Divide each side by 5.

Find each product.

2. -3(x + 5)**3.** 10(2a - b)1. 8(x + 2)

- 4. $4(v^2 4v + 9)$ 5. -3y(y-1)
- 6. $2r(-2r^2+6r-5)$ 7. 0.3(2p + 4)
- 9. $\frac{1}{2}(z+10)$ 8. 4.5 $(m^3 + m^2)$

Solve each equation.

11. -5(x + 8) = 5 **12.** 7(3s - 1) = -49**10.** 2(y + 3) = 10

14. 6(d + 1) - 6 = 1813. -4(-2w + 7) = -20

Practice

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Multiplying a Polynomial by a Monomial

Find each product. 1. 3(y + 4)**2.** -2(n + 3)**3.** 5(3a - 4)4. 7(-2c+3)5. x(x + 6)6. 8y(2y - 3)**9.** $6(a^2 + 5)$ 7. y(9 + 2y)8. -3b(b-1)**10.** -4m(-2 + 2m)11. -7n(-4n + 2)12. 2q(3q - 1)**13.** $p(3p^2 + 7)$ 14. $4x(5 - 2x^2)$ 15. $5b(b^2 + 5b)$ 16. $-3y(-9 + 3y^2)$ **17.** $2(8a^2 - 4a + 9)$ 18. $6(z^2 + 2z - 6)$ **19.** $x(x^2 - x + 3)$ **20.** $-4b(1-7b+b^2)$ **21.** $5m^2(3m^2 - m - 7)$ **22.** $-7y(-2 + 7y + 3y^2)$ **23.** $-3n^2(n^2 - 2n + 3)$ **24.** $9c(2c^3 + c^2 - 4)$ Solve each equation. **26.** 7(x-2) = -7**25.** 5(y + 2) = 25**27.** 2(a-5) + 4 = a + 9**28.** 3(2x + 6) - 10 = 4(x + 3)**29.** -6(2n-2) + 12 = 4(2n-9)**30.** b(b + 8) = b(b + 7) + 5**31.** y(y + 7) + 3y = y(y + 3) - 14**32.** m(m-5) + 14 = m(m+2) - 14

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Circular Areas and Volumes



Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)



Write an algebraic expression of the total volume of each figure.



Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.





Study Guide

Multiplying Binomials

The Distributive Property can be used to multiply binomials.

Example 1: Multiply (x + 2)(x + 5). (x + 2)(x + 5) = x(x + 5) + 2(x + 5)Distributive Property = x(x) + x(5) + 2(x) + 2(5)Distributive Property $= x^2 + 5x + 2x + 10$ Simplify. $= x^2 + 7x + 10$ Combine like terms.

Example 2: Multiply (3x + 1)(x - 4).

$$(3x + 1)(x - 4) = 3x(x - 4) + 1(x - 4)$$

 $= 3x(x) + 3x(-4) + 1(x) + 1(-4)$
 $= 3x^2 - 12x + x - 4$
 $= 3x^2 - 11x - 4$
Distributive Property
Distributive Property

You can also use a shortcut called the **FOIL method** to multiply two binomials. Find the four products indicated by the letters in the word FOIL. Then add the like terms.

Example 3: Multiply (y + 4)(y - 3).

F 0 T L First terms + Outer terms + Inner terms + Last terms (y + 4)(y - 3) =y(y) + y(-3)+4(y)+4(-3) $= v^2 - 3v + 4v - 12$ Add the like terms. $= v^2 + v - 12$

Find each product.

1.
$$(x + 6)(x + 3)$$
 2. $(y + 4)(y - 2)$

- **3.** (m 3)(m 1)4. (h - 10)(h + 7)
- 5. (w 8)(w 8)6. (g-2)(g+2)
- 7. (2a + 5)(a + 3)8. (p + 1)(2p + 3)
- 9. (3x 4)(x + 1)10. (z + 5)(4z - 3)

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Multiplying Binomials

Find each product. Use the Distributive Property or the FOIL method.

- **1.** (y + 4)(y + 3) **2.** (x + 2)(x + 1) **3.** (b + 5)(b 2)
- **4.** (a-6)(a-4) **5.** (z-5)(z+3) **6.** (n-1)(n-8)
- **7.** (x + 7)(x 4) **8.** (y 3)(y + 9) **9.** (b + 2)(b + 3)
- **10.** (2c + 5)(c 4) **11.** (4x 7)(x + 3) **12.** (x 1)(5x 4)
- **13.** (3y + 1)(3y + 2) **14.** (2n + 4)(5n 3) **15.** (7h 3)(4h 1)
- **16.** (2m 6)(3m + 2) **17.** (6a + 2)(2a + 3) **18.** (4c + 5)(2c 2)
- **19.** (x + y)(2x + y) **20.** (3a + 4b)(a 3b) **21.** (3m 3n)(3m 2n)
- **22.** (7p 4q)(2p + 3q) **23.** (2r + 2s)(2r + 3s) **24.** (3y 5z)(3y + 3z)
- **25.** $(x^2 + 1)(x 3)$ **26.** $(y 4) (y^2 + 2)$ **27.** $(2c^2 5)(c 4)$
- **28.** $(a^3 3a)(a + 4)$ **29.** $(b^2 + 2)(b^2 + 3)$ **30.** $(x^3 3)(4x + 1)$



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Powers of Binomials

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

- 2. Describe how to create the 6th row of Pascal's Triangle.
- **3.** Write the numbers for rows 6 through 10 of the triangle.
 - Row 6:
 - Row 7:
 - Row 8:
 - Row 9:
 - Row 10:

Multiply to find the expanded form of each product.

- 4. $(a + b)^2$
- 5. $(a + b)^3$
- 6. $(a + b)^4$

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

- 7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.
- 8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.

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Special Products

Recall x^2 means that x is used as a factor twice. Thus, $x^2 = x \cdot x$. When a binomial is squared, it is also used as a factor twice.

Therefore
$$(x + 3)^2 = (x + 3)(x + 3)$$
.
 $= x^2 + 3x + 3x + 9$ The inner and outer products are equal.
 $= x^2 + 6x + 9$
Similarly, $(x - 3)^2 = (x - 3)(x - 3)$.
 $= x^2 - 3x - 3x + 9$ The inner and outer products are equal.
 $= x^2 - 6x + 9$

Look at this special product of two binomials.

 $(x + 3)(x - 3) = x^2 + 3x - 3x - 9$ The inner and outer products are opposites. $= x^2 - 9$

Square of a Sum	Square of a Difference	Product of a Sum and a Difference
$(a + b)^2 = a^2 + 2ab + b^2$	$(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)(a - b) = a^2 - b^2$

Square of a difference

Replace a with r and b with 7.

6. $(1 + 5j)^2$

Example 1: Find $(r - 7)^2$.

$(a - b)^2 = a^2 - 2ab + b^2$	
$(r-7)^2 = r^2 - 2(r)(7) + 7^2$	
$= r^2 - 14r + 49$	

Example 2: Find (6y - 5)(6y + 5). $(a + b)(a - b) = a^2 - b^2$ Product of a sum and a difference $(6y - 5)(6y + 5) = (6y)^2 - 5^2$ Replace a with 6y and b with 5. $= 36y^2 - 25$

Find each product.

4. $(5b + 1)^2$

- 1. $(y + 8)^2$ 2. $(z - 4)^2$ 3. $(9 + a)^2$ 5. $(3d - e)^2$
- 8. (q + 7)(q 7)7. (x + 5)(x - 5)9. (m + 10)(m - 10)
- 11. (6x + 1)(6x 1)10. (k + 2)(k - 2)12. (2s + 3)(2s - 3)

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Special Pro	ducts		
Find each prod	luct.		
1. $(y + 4)^2$	2. $(x + 3)^2$	3. $(m + 6)^2$	
4. $(2b + c)^2$	5. $(x + 3y)^2$	6. $(4r + s)^2$	
7. $(2m + 2n)^2$	8. $(4a + 2b)^2$	9. $(3g + 3h)^2$	
10. $(b - 3)^2$	11. $(p - 4)^2$	12. $(s - 5)^2$	
13. $(3x - 3)^2$	14. $(2y - 3)^2$	15. $(c - 6d)^2$	
16. $(m - 2n)^2$	17. $(5x - y)^2$	18. $(a - 4b)^2$	
19. $(3p - 5q)^2$	20. $(2j - 4k)^2$	21. $(2r - 2s)^2$	
22. $(y + 3)(y - $	3) 23. $(x + 6)(x - 6)$	24. $(a + 9)(a + 3)$	- 9)
25. $(3a + b)(3a$	(4r + s)(4r - s)	27. $(2y + 6)(2)$	y — 6)
28. $(5x + 4)(5x$	(2c - 4) 29. $(2c + 4d)(2c - 4d)$	(3m + 6n) 30. $(3m + 6n)$	(3m - 6n)

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Squaring Numbers: A Shortcut

A shortcut helps you to square a positive two-digit number ending in 5. The method is developed using the idea that a two-digit number may be expressed as 10t + u. Suppose u = 5.

 $(10t + 5)^{2} = (10t + 5)(10t + 5)$ = 100t² + 50t + 50t + 25 = 100t² + 100t + 25 (10t + 5)² = 100t(t + 1) + 25

In words, this formula says that the square of a two-digit number has t(t + 1) in the hundreds place. Then 2 is the tens digit and 5 is the units digit.

Example: Using the formula for $(10t + 5)^2$, find 85^2 . $85^2 = 100 \cdot 8 \cdot (8 + 1) + 25$ = 7200 + 25 = 7225 Shortcut: First think $8 \cdot 9 = 72$. Then write 25.

Thus, to square a number, such as 85, you can write the product of the tens digit and the next consecutive integer t + 1. Then write 25.

Find each of the following using the shortcut.

1.	15^{2}	2.	25^{2}	3.	35^{2}
4.	45^{2}	5.	55^{2}	6.	65^{2}

Solve each problem.

- 7. What is the tens digit in the square of 95?8. What are the first two digits in the square of 75?
- **9.** Any three-digit number can be written as 100a + 10b + c. Square this expression to show that if the last digit of a three-digit number is 5 then the last two digits of the square of the number are 2 and 5.

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_____ Period

Buying in Bulk (Buyer)

NAME

Manufacturers often offer discounts to customers who buy in bulk. A clock manufacturer, for example, might sell one clock movement for \$14.95 but drop the price if many clock movements are ordered at once.

School-to-Workplace

Store buyers are trained to look for quality in the merchandise they purchase. In fact, there are courses in junior colleges that focus on merchandising. In addition to looking for quality, buyers also keep an eye out for bargains. One such bargain will be the purchase of goods in bulk.

The expression below gives an idea of how the individual price in dollars declines when a buyer purchases *x* clock movements for all $x \leq 750$.

14.95 - 0.01(x - 1)

Find an expression that represents the total cost of a purchase of x units. Then use the expression to find the cost of purchasing 200 units.

Simplify x[14.95 - 0.01(x - 1)].

x[14.95 - 0.01(x - 1)] = x[14.95 - 0.01x + 0.01)] $= 14.95x - 0.01x^2 + 0.01x$ Distributive Property $= 14.96x - 0.01x^{2}$

If x = 200, then the total cost is given by the expression below.

 $14.96 \times 200 - 0.01 \times 200^2 = 2590$

The total cost of the bulk purchase is \$2590.

Solve.

- 1. If the cost of purchasing one unit is 3.89 0.005(x 1), find an expression for the total cost of x units. Then use the expression to find the cost of purchasing 300 units.
- 2. If the cost of buying one unit is 13.02 0.002(x 1), find an expression for the total cost of x units. Then use the expression to find the cost of purchasing 400 units.
- 3. If the cost of purchasing one unit is 2.22 0.0015(x 1), find an expression for the total cost of x units. Then use the expression to find the cost of purchasing 275 units.

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